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QUARK-ANTIQUARK INTERACTION AND CHIRAL SYMMETRY

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ABSTRACT

To explain the structure of both light and heavy hadrons in terms of quarks, it is necessary to use a scalar confining potential to reproduce the heavy meson spectrum, and at the same time to implement spontaneous breaking of chiral symmetry which is important in the case of light hadrons. We investigate the constraints imposed to the Lorentz structure of the quark-antiquark interaction by chiral symmetry and partial conservation of the axial current.

1. Introduction

It has been a major achievement of potential models to reproduce the heavy meson spectrum very accurately. The Lorentz structure of the confining interaction which gives the best results is a scalar one [1]. In the light sector, potential models fail to reproduce the small masses of pseudoscalar mesons such as the pion. This has revived some interest on chiral models. The quark axial current is partially conserved: in the chiral limit, when the quark mass approaches zero, the current is conserved, and the pion mass becomes zero. Another important feature of chiral models is the spontaneous breakdown of chiral symmetry and the existence of a chiral condensate.

If one wants to explain the structure of both light and heavy hadrons, one needs a confining potential and also to ensure partial conservation of axial current (PCAC). A phenomenological confining interaction among quarks and antiquarks must have the general form [2]

$$\widehat{K} = K_s + K_p\gamma_5 \otimes \gamma_5 + K_v\gamma_\mu \otimes \gamma^\mu + K_a\gamma_5\gamma_\mu \otimes \gamma_5\gamma^\mu + \frac{1}{2}K_t\sigma_{\mu,\nu} \otimes \sigma^{\mu,\nu}, \tag{1}$$

where the indices stand for scalar, pseudoscalar, vector, axial-vector and tensor respectively, and the potentials K_i are all functions of the four-momentum q (ladder approximation). In order to obtain a potential model, we assume an instantaneous interaction, which implies that $\widehat{K}$ does not depend on p_o. To see the consequences of PCAC, we will use Ward identities.

2. Ward identities and the gap equation

The vertex function, Γ_μ, associated to the vector current must satisfy the equation

$$\Gamma_\mu(p,p') = \gamma_\mu + i\int \frac{d^4q}{(2\pi)^4}\widehat{K}(q)S(p'+q)\Gamma_\mu(p'+q,p+q)S(p+q). \tag{2}$$

Conservation of the vector current can be expressed in the form of a Ward identity

$$i(p-p')^\mu\Gamma_\mu(p',p) = S^{-1}(p') - S^{-1}(p). \tag{3}$$

These two equations can be solved to give an integral equation for the propagator S in terms of the interaction kernel $\widehat{K}$ (gap equation) [3]. We can now use a similar teatment for the axial current. There are two axial vertex functions Γ^5_μ and Γ^5 which must satisfy the equations

$$\Gamma^5_\mu(p,p') = \gamma_\mu\gamma_5 + i\int \frac{d^4q}{(2\pi)^4}\widehat{K}(q)S(p'+q)\Gamma^5_\mu(p'+q,p+q)S(p+q), \tag{4}$$

$$\Gamma^5(p,p') = \gamma_5 + i\int \frac{d^4q}{(2\pi)^4}\widehat{K}(q)S(p'+q)\Gamma^5(p'+q,p+q)S(p+q). \tag{5}$$

Partial conservation of the axial current implies the Ward identity

$$i(p-p')^\mu\Gamma^5_\mu(p',p) + i2m\Gamma^5(p',p) = \gamma_5 S^{-1}(p) + S^{-1}(p')\gamma_5\ , \tag{6}$$

which is valid for any quark mass m. Solving the last three equations, a gap equation is also obtained, for the propagator S, in terms of $\widehat{K}$ [3]. With the local potentials we use, it is found that for certain choices of the interaction kernel the gap equations obtained from the vector and axial currents, do not have a common solution for the propagator S. However, if the interaction kernel satisfies certain conditions, the two equations become equal and admit a common solution. The necessary and sufficient conditions are [4]

$$K_s(q) = K_p(q) = -3K_t(q). \tag{7}$$

Namely, the interaction kernel must have the form *scalar* + *pseudoscalar* - *tensor*/3. The vector and axial terms are not constrained, but they are the only two that enter into the gap equation, if Eq. 7 is imposed.

Any interaction that does not follow the conditions given above, will not lead to a quark (antiquark) propagator S that satisfies both vector current conservation and PCAC. It is important to notice that these constraints are independent of the quark mass, even though the term that violates conservation of the axial current is very large for heavy quarks.

3. Quark mass and energy in the condensed medium

With an instantaneous interaction, $K(\mathbf{q})$, the quark propagator can be written in the form

$$S(p) = i\left[\gamma_0 p_0 - E\sin\phi(\hat{\mathbf{p}}\cdot\boldsymbol{\gamma}) - E\cos\phi\right]^{-1} , \tag{8}$$

where the functions ϕ and E depend only on $|\mathbf{p}|$. If we write

$$\tan\phi = |\mathbf{p}|/m^\star , \tag{9}$$

we can then calculate the functions $m^\star$ and $E(|\mathbf{p}|)$ from the gap equation, as a function of the potential. If the vector and axial terms of the interaction are zero, and Eq. 7 holds —which includes the case of free quarks— $m^\star$ becomes constant and equal to the quark mass m_o, and E is the relativistic energy of the quark: $E_o^2 = |\mathbf{p}|^2 + m_o^2$.

If the confining interaction includes a vector and (or) axial term, the solution of the gap equation gives $m^\star$ depending on momentum, and an effective energy E. These effective mass and energy can be explained as the effect of chiral-symmetry breaking, which leads to a condensed vacuum [5,6] —the so-called *chiral condensate.*

For very massive quarks $m^\star$ is almost constant and equal to m_o, and the effective energy is very similar to the relativistic energy. In the case of the *up* and *down* quarks, the strength of a typical confining potential is much larger than their masses, and the effective mass $m^\star$ varies from a large value at $|\mathbf{p}| = 0$, down to the current mass m_o at large momentum. The effective energy of the light quarks departs significantly from the relativistic energy, at small momentum, and it can even become negative.

4. Meson spectrum

The bound states of quark-antiquark are obtained from the Salpeter equation. For heavy flavors, the semi-relativistic limit leads to the usual Hamiltonian used in potential models, and known as the generalized Breit-Fermi Hamiltonian [2]. It has spin-spin, spin-orbit and tensor terms that are not imposed arbitrarily, but directly derived from the Lorentz structure of the quark-antiquark interaction kernel.

According to the conditions we derived in the previous section, the confining part of the interaction kernel cannot be purely scalar as usually assumed in potential models. It must include a pseudoscalar term with the same functional form, and a tensor term equal to $-1/3$ times the scalar term. In addition to that, we need vector or axial terms if we want to generate a chiral condensate and large constituent masses for the light quarks. In a previous paper we have looked at the case of a quadratic confining potential, and we have shown that these constraints can be implemented, without loosing the good results for the heavy meson mass-spectrum [4].

In the light-quark sector, the generalized Breit-Fermi Hamiltonian cannot be used, mainly because of two reasons. First, the non-relativistic limit for the quark energy is not a good approximation; this can be amended by relativistic corrections to the kinetic energy. But even with relativistic corrections the second problem arises from the fact that the quark propagator is modified inside the chiral condensate and one has to use the effective mass m^*, and an effective quark energy that can become negative at small momentum. This has been done successfully by some authors, in the case of a quadratic, vector confining- potential [5,6]. The result are pseudoscalar mesons with small masses which can be finely tuned to the experimental values, by small changes in the current-quark mass m_o. In the chiral limit, $m_o = 0$, the pseudoscalar mesons are massles as a consequence of the negative effective energy. However, with a purely vector kernel the results for heavy mesons will not be so good.

5. Conclusions

We have studied a model which has confinement and leads to a chiral condensate vacuum. A preliminary study in the case of a quadratic confining potential seems to indicate that the model will give satisfactory results, with a more realistic linear potential. The strong point of this model is that it gives a unified description of both light and heavy hadrons, in terms of current quarks, by incorporating confinement and spontaneous chiral-symmetry breaking.

6. References

1. A. B. Henriques, B. H. Kellet and R. G. Moorhouse, *Phys. Lett.* **64B** (1976) 85.

2. D. Gromes, *Nucl. Phys. B* **207** (1977) 80.

3. S. L. Adler and A. C. Davis, *Nucl. Phys. B* **244** (1984) 469.

4. J. E. Villate, D-S. Liu, J. E. Ribeiro and P.J.A. Bicudo, *Phys. Rev. D* **47** (1992) 1145.

5. A. Le Yaouanc, L. Oliver, S. Ono, O. Pène and J-C. Raynal, *Phys. Rev. D* **31** (1985) 137.

6. P. J. de A. Bicudo, J.E.F.T. Ribeiro, *Phys. Rev. D* **42** (1991) 1611, 1625.