Dinâmica e Sistemas Dinâmicos – Formulário

1. Cinemática

$$\bar{v} = \frac{\Delta s}{\Delta t} \qquad \bar{a}_\mathrm{t} = \frac{\Delta v}{\Delta t} \qquad v = \frac{\mathrm{d}\,s}{\mathrm{d}\,t} \qquad a_\mathrm{t} = \frac{\mathrm{d}\,v}{\mathrm{d}\,t} \qquad a_\mathrm{t} = v\,\frac{\mathrm{d}\,v}{\mathrm{d}\,s}$$

2. Cinemática vetorial

$$v_x = \frac{\mathrm{d}\,x}{\mathrm{d}\,t} \quad (\text{ou } y \text{ ou } z) \qquad a_x = \frac{\mathrm{d}\,v_x}{\mathrm{d}\,t} \quad (\text{ou } y \text{ ou } z) \qquad a_x = v_x\,\frac{\mathrm{d}\,v_x}{\mathrm{d}\,x} \quad (\text{ou } y \text{ ou } z)$$

$$\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k} \qquad \vec{v} = \frac{\mathrm{d}\,\vec{r}}{\mathrm{d}\,t} \qquad \vec{a} = \frac{\mathrm{d}\,\vec{v}}{\mathrm{d}\,t} \qquad \vec{r} = \vec{r}_i + \int\limits_{t_i}^{t} \vec{v}(t')\,\mathrm{d}\,t' \qquad \vec{v} = \vec{v}_i + \int\limits_{t_i}^{t} \vec{a}(t')\,\mathrm{d}\,t'$$

Movimento relativo: $\vec{r}_\mathrm{P} = \vec{r}_\mathrm{P/Q} + \vec{r}_\mathrm{Q}$ $\qquad \vec{v}_\mathrm{P} = \vec{v}_\mathrm{P/Q} + \vec{v}_\mathrm{Q}$ $\qquad \vec{a}_\mathrm{P} = \vec{a}_\mathrm{P/Q} + \vec{a}_\mathrm{Q}$

Produto escalar: $\vec{a}\cdot\vec{b} = a\,b\cos\theta$ $\qquad \vec{a}\cdot\vec{b} = a_x\,b_x + a_y\,b_y + a_z\,b_z$ $\qquad a = \sqrt{\vec{a}\cdot\vec{a}}$

3. Movimento curvilíneo

$$\vec{v} = \dot{s}\,\hat{e}_\mathrm{t} \qquad \vec{a} = \dot{v}\,\hat{e}_\mathrm{t} + \frac{v^2}{R}\,\hat{e}_\mathrm{n} \qquad a^2 = a_\mathrm{t}^2 + a_\mathrm{n}^2$$

Movimento circular: $s = R\,\theta$ $\qquad v = R\,\omega$ $\qquad a_\mathrm{t} = R\,\alpha$

Produto vetorial: $\vec{a}\times\vec{b} = a\,b\sin\theta\,\hat{n}$ $\qquad \vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$ $\qquad \vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$

Rotação dos corpos rígidos: $\vec{v} = \vec{\omega}\times\vec{r}$ $\qquad \vec{\alpha} = \dfrac{\mathrm{d}\vec{\omega}}{\mathrm{d}\,t}$ $\qquad \vec{a} = \vec{\alpha}\times\vec{r} + \vec{\omega}\times\vec{v}$

Rotação plana: $v_\mathrm{b/a} = R_\mathrm{b/a}\,\omega$ $\qquad \vec{\omega} = \omega\,\hat{e}_\mathrm{eixo}$ $\qquad \omega = \dfrac{\mathrm{d}\theta}{\mathrm{d}\,t}$ $\qquad \alpha = \dfrac{\mathrm{d}\omega}{\mathrm{d}\,t}$ $\qquad \alpha = \omega\,\dfrac{\mathrm{d}\omega}{\mathrm{d}\theta}$

4. Mecânica vetorial

$$\vec{p} = m\,\vec{v} \qquad \int\limits_{t_1}^{t_2} \vec{F}\,\mathrm{d}\,t = \vec{p}_2 - \vec{p}_1 \qquad \vec{F} = m\,\vec{a} \qquad \vec{P} = m\,\vec{g} \qquad F_\mathrm{e} \le \mu_\mathrm{e}\,R_\mathrm{n} \qquad F_\mathrm{c} = \mu_\mathrm{c}\,R_\mathrm{n}$$

Esfera num fluido: $N_\mathrm{R} = r\,v\left(\dfrac{\rho}{\eta}\right)$ $\qquad F_\mathrm{f} = 6\pi\,\eta\,r\,v \quad (N_\mathrm{R} < 1)$ $\qquad F_\mathrm{f} = \dfrac{\pi}{4}\,\rho\,r^2\,v^2 \quad (N_\mathrm{R} > 10^3)$

5. Dinâmica dos corpos rígidos

$$M_\mathrm{o} = F\,b \qquad \vec{M}_\mathrm{o} = \vec{r}\times\vec{F} \qquad M_z = \begin{vmatrix} x & y \\ F_x & F_y \end{vmatrix} \qquad \vec{r}_\mathrm{cm} = \frac{1}{m}\int \vec{r}\,\mathrm{d}\,m \qquad \vec{v}_\mathrm{cm} = \frac{1}{m}\int \vec{v}\,\mathrm{d}\,m$$

$$\vec{a}_\mathrm{cm} = \frac{1}{m}\int \vec{a}\,\mathrm{d}\,m \qquad \sum_{i=1}^{n} \vec{F}_i = m\,\vec{a}_\mathrm{cm} \qquad \sum_{i=1}^{n} M_{z,i} = I_z\,\alpha \qquad I_z = \int R^2\,\mathrm{d}\,m$$

6. Trabalho e energia

$$W_{12} = \int\limits_{s_1}^{s_2} F_\mathrm{t}\,\mathrm{d}\,s \qquad W_{12} = E_\mathrm{c}(2) - E_\mathrm{c}(1) \qquad E_\mathrm{c} = \frac{1}{2}m\,v_\mathrm{cm}^2 + \frac{1}{2}I_\mathrm{cm}\,\omega^2 \qquad U = -\int\limits_{\vec{r}_0}^{\vec{r}} \vec{F}\cdot\mathrm{d}\,\vec{r} \qquad W_{12} = U(1) - U(2)$$

$$U_\mathrm{g} = m\,g\,z \qquad U_\mathrm{e} = \frac{1}{2}k\,s^2 \qquad E_\mathrm{m} = E_\mathrm{c} + U \qquad \int\limits_{s_1}^{s_2} F_\mathrm{t}^\mathrm{nc}\,\mathrm{d}\,s = E_\mathrm{m}(2) - E_\mathrm{m}(1)$$

Oscilador harmónico simples: $\Omega = \sqrt{\dfrac{k}{m}} = 2\pi\,f$ $\qquad s = A\sin(\Omega\,t + \phi_0)$ $\qquad E_\mathrm{m} = \dfrac{1}{2}m\,v^2 + \dfrac{1}{2}k\,s^2$

7. Sistemas dinâmicos

$$\dot{x}_1 = f_1(x_1, x_2) \qquad \dot{x}_2 = f_2(x_1, x_2) \qquad \vec{u} = f_1(x_1, x_2)\,\hat{e}_1 + f_2(x_1, x_2)\,\hat{e}_2$$

Equações diferenciais de segunda ordem: $\ddot{x} = f(x, \dot{x}) \qquad y = \dot{x} \qquad \vec{u} = y\,\hat{\imath} + f(x, y)\,\hat{\jmath}$

Sistemas conservativos: $\dfrac{\partial f_1}{\partial x_1} + \dfrac{\partial f_2}{\partial x_2} = 0 \qquad f_1 = \dfrac{\partial H}{\partial x_2} \qquad f_2 = -\dfrac{\partial H}{\partial x_1}$ Evolução: $H =$ constante

8. Mecânica lagrangiana

$$\frac{\mathrm{d}}{\mathrm{d}\,t}\left(\frac{\partial E_\mathrm{c}}{\partial \dot{q}_j}\right) - \frac{\partial E_\mathrm{c}}{\partial q_j} + \frac{\partial U}{\partial q_j} = Q_j \qquad Q_j = \sum_i \vec{F}_i \cdot \frac{\partial \vec{r}_i}{\partial q_j}$$

Multiplicadores de Lagrange:

$$\frac{\mathrm{d}}{\mathrm{d}\,t}\left(\frac{\partial E_\mathrm{c}}{\partial \dot{q}_j}\right) - \frac{\partial E_\mathrm{c}}{\partial q_j} + \frac{\partial U}{\partial q_j} - \lambda\,\frac{\partial f}{\partial q_j} = Q_j \qquad \lambda\,\frac{\partial f}{\partial q_j} = \text{comp. } j \text{ da força/momento de ligação}$$

9. Sistemas lineares

$$\frac{\mathrm{d}\,\vec{r}}{\mathrm{d}\,t} = \mathbf{A}\,\vec{r} \qquad \vec{r} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad \mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \lambda^2 - \mathrm{tr}(\mathbf{A})\,\lambda + \det(\mathbf{A}) = 0$$

Valores próprios λ	Tipo de ponto	Tipo de equilíbrio
2 reais; sinais opostos	ponto de sela	instável
2 reais, positivos	nó repulsivo	instável
2 reais, negativos	nó atrativo	estável
2 complexos; parte real positiva	foco repulsivo	instável
2 complexos; parte real negativa	foco atrativo	estável
2 imaginários	centro	estável
1 real, positivo	nó impróprio	instável
1 real, negativo	nó impróprio	estável

10. Sistemas não lineares

$$\dot{x}_1 = f_1(x_1, x_2) \qquad \dot{x}_2 = f_2(x_1, x_2) \qquad (f_1 \text{ e } f_2 \text{ funções contínuas}) \qquad \mathbf{J}(x_1, x_2) = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix}$$

Pêndulo: $\ddot{\theta} = -\dfrac{g}{l}\sin\theta \qquad l = \dfrac{r_\mathrm{g}^2}{r_\mathrm{cm}}$

11. Ciclos limite e dinâmica populacional

Sistemas de duas espécies:

$$\dot{x}_1 = f_1(x_1, x_2) \qquad \dot{x}_2 = f_2(x_1, x_2) \qquad \lim_{x_1 \to 0} f_1(x_1, x_2) = 0 \qquad \lim_{x_2 \to 0} f_2(x_1, x_2) = 0$$

Sistema com cooperação: $\dfrac{\partial f_1}{\partial x_2}$ e $\dfrac{\partial f_2}{\partial x_1}$ positivas.

Sistema com competição: $\dfrac{\partial f_1}{\partial x_2}$ e $\dfrac{\partial f_2}{\partial x_1}$ negativas.

Sistema predador presa: $\dfrac{\partial f_1}{\partial x_2}$ e $\dfrac{\partial f_2}{\partial x_1}$ com sinais opostos.