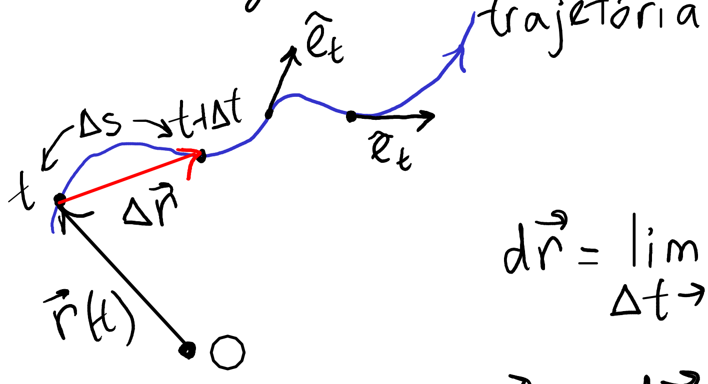
aula 5. 8 de março

MOVIMENTO CURVILINEO

versor tangencial $\hat{e}_t$



deslocamento no intervalo $[t, t+\Delta t]$:

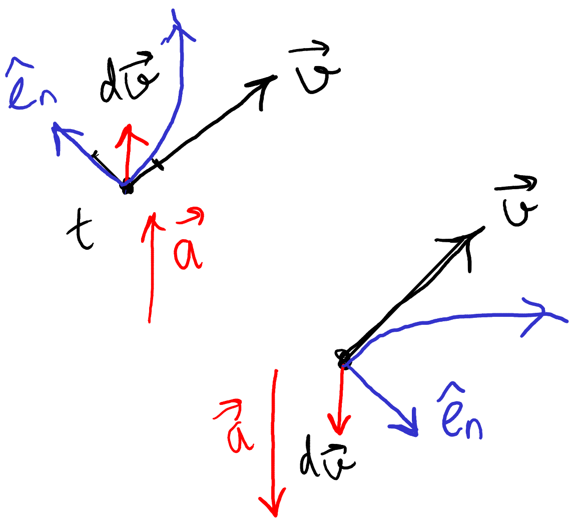
$$\Delta\vec{r} = \vec{r}(t+\Delta t) - \vec{r}(t)$$

$$d\vec{r} = \lim_{\Delta t \to 0} \Delta\vec{r} = ds\,\hat{e}_t$$

$$\Rightarrow \vec{v} = \frac{d\vec{r}}{dt} = \frac{ds}{dt}\hat{e}_t \qquad \boxed{\vec{v} = v\hat{e}_t}$$

numa trajetória fixa, v pode ser negativa

versor normal $\hat{e}_n$



$d\vec{v}$ = variação de $\vec{v}$ num intervalo infinitesimal dt

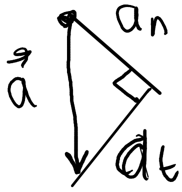
$$d\vec{v} = \vec{a}\,dt$$

$$\boxed{\vec{a} = a_t\hat{e}_t + a_n\hat{e}_n}$$

aceleração tangencial
- positiva (aumenta $|\vec{v}|$)
- negativa (diminui $|\vec{v}|$)

aceleração normal (>0)

$|\vec{v}|$ = rapidez



$$\boxed{|\vec{a}|^2 = a_t^2 + a_n^2}$$

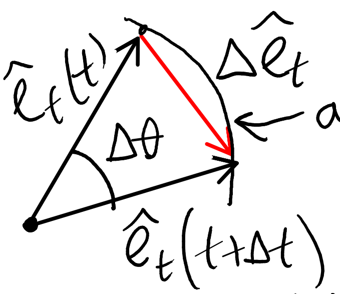
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d(v\hat{e}_t)}{dt} = \frac{dv}{dt}\hat{e}_t + v\frac{d\hat{e}_t}{dt}$$

Derivada do versor tangencial

$$\hat{e}_t \cdot \hat{e}_t = 1 \quad \Rightarrow \frac{d(\hat{e}_t \cdot \hat{e}_t)}{dt} = 0$$

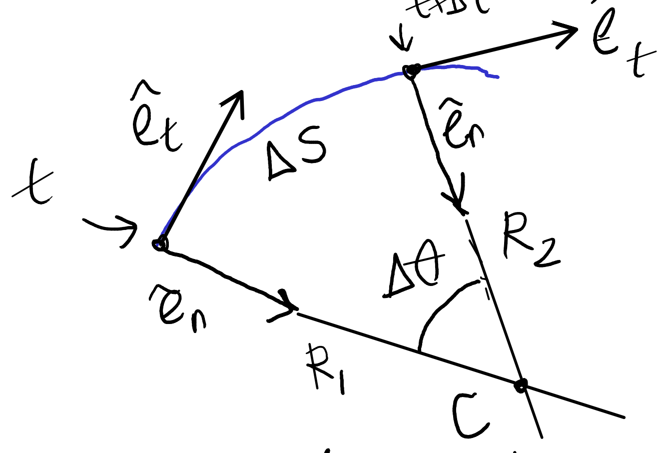
$$\frac{d\hat{e}_t}{dt} \cdot \hat{e}_t + \hat{e}_t \cdot \frac{d\hat{e}_t}{dt} = 2\hat{e}_t \cdot \frac{d\hat{e}_t}{dt} \quad \Rightarrow \hat{e}_t \cdot \frac{d\hat{e}_t}{dt} = 0$$

$$\begin{cases} \dfrac{d\hat{e}_t}{dt} = 0 \rightarrow \text{movimento retilíneo } (\vec{a} = \dot{v}\hat{e}_t) \\ \dfrac{d\hat{e}_t}{dt} \text{ perpendicular a } \hat{e}_t \rightarrow \text{movimento curvilíneo} \end{cases}$$



arco de circ. com raio 1

$$\frac{d\hat{e}_t}{dt} = \lim_{\Delta t \to 0} \frac{\Delta\hat{e}_t}{\Delta t} = d\theta\,\hat{e}_n$$



$\Delta t \to 0$: $R_1 \to R_2 = R$

R = raio de curvatura da trajetória

C = centro local da trajetória

$$\Delta s \rightarrow ds = R\,d\theta$$

$$d\hat{e}_t = \frac{ds}{R}\,\hat{e}_n \Rightarrow \frac{d\hat{e}_t}{dt} = \frac{\dot{s}}{R}\,\hat{e}_n = \frac{v}{R}\,\hat{e}_n$$

$$\boxed{\vec{a} = \dot{v}\,\hat{e}_t + \frac{v^2}{R}\,\hat{e}_n}$$

$$a_t = \dot{v}$$

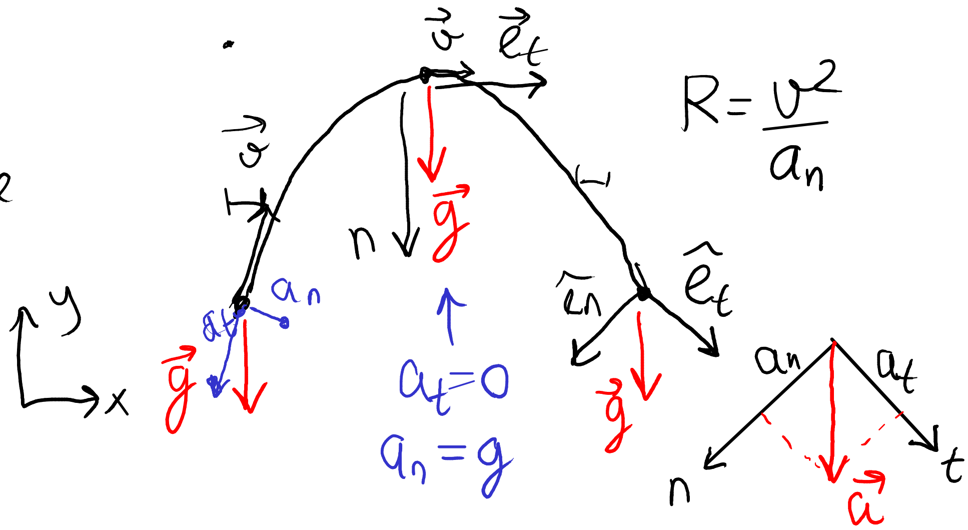
$$a_n = \frac{v^2}{R}$$

Exemplos:

lançamento de projéteis

$$a_t = \frac{\vec{a}\cdot\vec{v}}{v}$$

$$R = \frac{v^2}{a_n}$$



$$\vec{v}(t) = v_{0x}\,\hat{\imath} + (v_{0y} - gt)\,\hat{\jmath}$$

$$\left(\vec{a} = -g\hat{\jmath} \rightarrow \vec{v} = \vec{v}_0 - \int_0^t g\hat{\jmath}\,dt\right)$$

Exemplo 3.1. $\vec{r}(t) = 5t\,\hat{\imath} + \frac{3}{2}t^2\,\hat{\jmath} + 2(1-t^2)\,\hat{k}$ (SI)

Determine: ⓐ $v(t)$ ⓑ $R(t)$ ⓒ ΔS no intervalo $t \in [0,1]$

ⓐ $\vec{v} = \frac{d\vec{r}}{dt} = 5\hat{\imath} + 3t\,\hat{\jmath} - 4t\,\hat{k}$

$$v(t) = +\sqrt{\vec{v}\cdot\vec{v}} = \sqrt{25 + 9t^2 + 16t^2} = 5\sqrt{t^2+1}$$

(b) $R = \dfrac{v^2}{a_n}$ $\qquad \vec{a} = \dfrac{d\vec{v}}{dt} = 3\hat{j} - 4\hat{k}$ $\qquad |\vec{a}| = \sqrt{9+16} = 5$

$$a_t = \frac{dv}{dt} = 5\left(\frac{1}{\cancel{2}}(t^2+1)^{-1/2}\cancel{2}t\right) = \frac{5t}{\sqrt{t^2+1}}$$

$\left(\text{ou: } a_t = \dfrac{\vec{a}\cdot\vec{v}}{v}\right)$ $\qquad a_n = \sqrt{|\vec{a}|^2 - a_t^2}$

$$a_n = \sqrt{25 - \frac{25t^2}{t^2+1}} = 5\sqrt{\frac{t^2+1-t^2}{t^2+1}} = \frac{5}{\sqrt{t^2+1}}$$

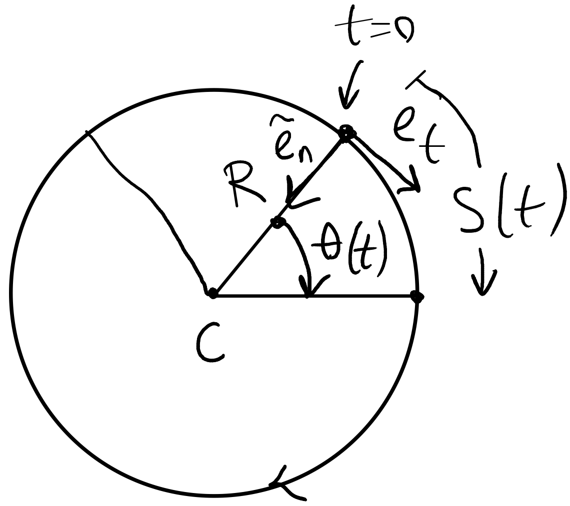
$$R(t) = \frac{25(t^2+1)}{5(t^2+1)^{-1/2}} = 5(t^2+1)^{3/2}$$

(c) $v = \dfrac{ds}{dt} = 5\sqrt{t^2+1}$ $\qquad \displaystyle\int_0^{\Delta s} ds = 5\int_0^1 \sqrt{t^2+1}\, dt$

$\Delta s \approx 5.739$ m

MOVIMENTO CIRCULAR

$\begin{cases} \text{centro C fixo} \\ \text{R constante} \end{cases}$



$s(t) = R\,\theta(t)$

$v = \dot{s} = R\dot{\theta}$ $\boxed{v = R\omega}$ $\omega = \dot{\theta}$ = velocidade angular

$a_t = \dot{v} = R\dot{\omega}$ $\boxed{a_t = R\alpha}$ $\alpha = \dot{\omega}$ = aceleração angular

$\theta(t)$: $\underbrace{\omega = \dot{\theta}, \quad \alpha = \dot{\omega}, \quad \alpha = \omega \frac{d\omega}{d\theta}}_{\text{equações cinemáticas}}$

$\left(v = \dot{s}, \quad a_t = \dot{v}, \quad a_t = v\frac{dv}{ds} \right)$

movimento circular uniforme $\alpha = 0$

$\Rightarrow \omega = \text{constante}$

$\theta(t) = \theta_0 + \omega t$ (aumenta 2π num tempo $t = T$)

período de rotação: $T \rightarrow \omega T = 2\pi$

$$T = \frac{2\pi}{\omega}$$