POTENCIAL ELETROSTÁTICO

$\overset{P}{\cdot}$ V_P $\qquad$ $\overset{Q}{\circ}$ V_Q

V : função da posição

cada ponto do espaço tem um valor de V

$$V_Q - V_P = -\int_P^Q \vec{E}\cdot d\vec{r} \quad \text{(qualquer percurso de integração)}$$

$\vec{E}$ é conservativo $\int_P^Q \vec{E}\cdot d\vec{r}$ não depende do percurso de P até Q

Seja: $P = (x, y, z)$ e $Q = (x+\Delta x, y, z)$ e $d\vec{r} = \hat{\imath}\, dx$ (reta de P até Q)

$$V_Q - V_P = -\int_{(x,y,z)}^{(x+\Delta x, y, z)} (\vec{E}\cdot\hat{\imath})\, dx = -\int_{(x,y,z)}^{(x+\Delta x,y,z)} E_x\, dx = -\bar{E}_x\, \Delta x$$

valor médio

$$\bar{E}_x = -\frac{V(x+\Delta x, y, z) - V(x,y,z)}{\Delta x}$$

$$E_x(x,y,z) = \lim_{\Delta x \to 0} -\frac{V(x+\Delta x, y, z) - V(x,y,z)}{\Delta x} = -\frac{\partial V}{\partial x}$$

derivada parcial (com y e z constantes)

Da mesma forma: $E_y = -\dfrac{\partial V}{\partial y}$, $E_z = -\dfrac{\partial V}{\partial z}$

$$\vec{E}(x,y,z) = -\frac{\partial V}{\partial x}\hat{\imath} - \frac{\partial V}{\partial y}\hat{\jmath} - \frac{\partial V}{\partial z}\hat{k} = -\text{gradiente de } V$$

$V(x,y,z)$ deverá ser função contínua.

OPERADOR NABLA

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\Updownarrow$$

$\vec{E}$ é conservativo

$$\Updownarrow$$

$$\frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x} \quad \left(= -\frac{\partial^2 V}{\partial x \partial y} = -\frac{\partial^2 V}{\partial y \partial x} \right)$$

$$\frac{\partial E_y}{\partial z} = \frac{\partial E_z}{\partial y} \quad \left(= -\frac{\partial^2 V}{\partial y \partial z} = -\frac{\partial^2 V}{\partial z \partial y} \right)$$

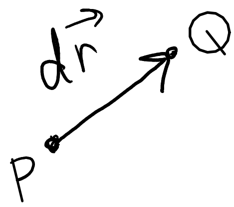
$$\frac{\partial E_x}{\partial z} = \frac{\partial E_z}{\partial x} \quad \left(= -\frac{\partial^2 V}{\partial z \partial x} = -\frac{\partial^2 V}{\partial x \partial z} \right)$$

$\Leftrightarrow \quad \vec{\nabla} \times \vec{E} = \vec{0}$ (rotacional de $\vec{E}$)

$$\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \times \left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right) = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\hat{i} + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\hat{j} + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\hat{k}$$

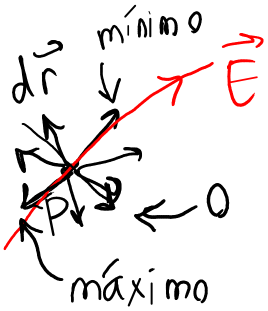
$$= 0\hat{i} + 0\hat{j} + 0\hat{k} = \vec{0}$$ ← em qualquer ponto P

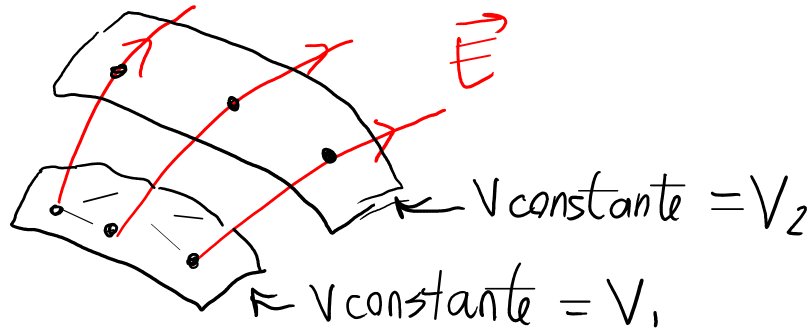
SUPERFÍCIES EQUIPOTENCIAIS



$$V_Q - V_P = dV = -\vec{E} \cdot d\vec{r}$$

$$dV = -\vec{E}\cdot d\vec{r} = \begin{cases} 0, \text{ se } \vec{E} \text{ for perpendicular a } d\vec{r} \\ \text{máximo, se } \vec{E} \text{ for na direção de } d\vec{r}\text{, no sentido oposto} \\ \text{mínimo, se } \vec{E} \text{ for na direção e sentido de } d\vec{r} \end{cases}$$

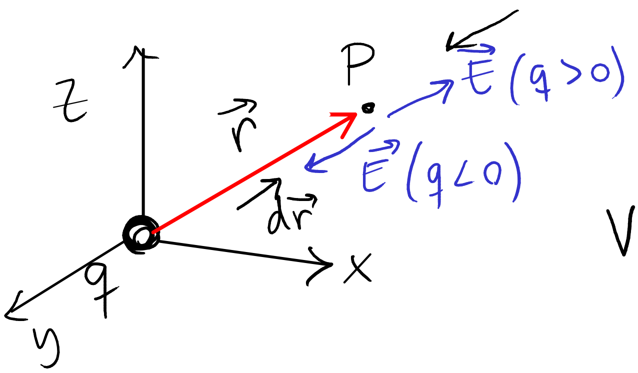




$V_1 > V_2$

Superfícies equipotenciais = superfícies perpendiculares às linhas de campo $\vec{E}$

POTENCIAL DE CARGAS PONTUAIS



arbitra-se $V=0$ se $r\to\infty$

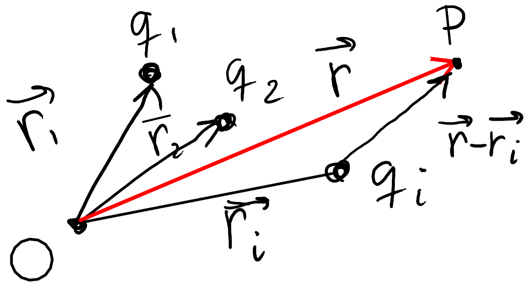
$$V_P = -\int_{\infty}^{P} \vec{E}\cdot d\vec{r} \quad \text{(usa-se o percurso radial)}$$

$$\vec{E}\cdot d\vec{r} = \pm\left(\frac{k|q|}{r^2}\right)dr = \frac{kq}{r^2}\,dr$$

$$V(x,y,z) = -\int_{\infty}^{|\vec{r}|} \frac{kq}{r^2}\,dr = +\frac{kq}{r}\Bigg|_{r\to\infty}^{r} = \frac{kq}{r}$$

$$\boxed{V(x,y,z) = \frac{kq}{\sqrt{x^2+y^2+z^2}}}$$

n cargas pontuais.



$$V(\vec{r}) = \sum_{i=1}^{n} \frac{k q_i}{|\vec{r} - \vec{r}_i|}$$

Caso particular: cargas num plano (plano xy)

potencial no plano xy $(\vec{r} = x\hat{\imath} + y\hat{\jmath})$

$$\boxed{V(x,y) = \sum_{i=1}^{n} \frac{k q_i}{\sqrt{(x-x_i)^2 + (y-y_i)^2}}}$$

Exemplo. Determine o potencial, no plano xy, produzido pelas seguintes 3 cargas (no plano xy):

carga (μC)	x_i (cm)	y_i (cm)
−4	20	10
+3	10	−30
+2	−30	0

Resolução: unidades → $q \to \mu C$, $(x,y) \to dm$

$$k = 9\times10^9 \frac{N\cdot m^2}{C^2} = 9\times10^9 \frac{V\cdot m}{C} = 9\times10^9 \frac{V\cdot(10\,dm)}{10^6\,\mu C}$$

$$\boxed{k = 90 \frac{kV\cdot dm}{\mu C}}$$

$q_1 = -4$ $\quad \vec{r}_1 = (2,1)$

$q_2 = +3$ $\quad \vec{r}_2 = (1,-3)$

$q_3 = +2$ $\quad \vec{r}_3 = (-3,0)$

$$V(x,y) = -\frac{360}{\sqrt{(x-2)^2+(y-1)^2}} + \frac{270}{\sqrt{(x-1)^2+(y+3)^2}} + \frac{180}{\sqrt{(x+3)^2+y^2}}$$

No Maxima:

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File  Edit  View  Search  Terminal  Help
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
(%i1) norma(v) := sqrt(v.v)$

(%i2) q: [-4,3,2]$

(%i3) p: [[2,1],[1,-3],[-3,0]]$

(%i4) V: sum(90*q[i]/norma([x,y]-p[i]),i,1,3);
                 270                         180
(%o4) ------------------------- + -------------------
                  2           2           2          2
      sqrt((y + 3)  + (x - 1) )   sqrt(y  + (x + 3) )
                                                      360
                                              - -------------------------
                                                         2           2
                                                 sqrt((y - 1)  + (x - 2) )
(%i5) ploteq(V,[x,-5,5],[y,-5,5])$

(%i6) ploteq(V,[x,-50,50],[y,-50,50])$

(%i7) ploteq(V,[x,-500,500],[y,-500,500])$

(%i8) 
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