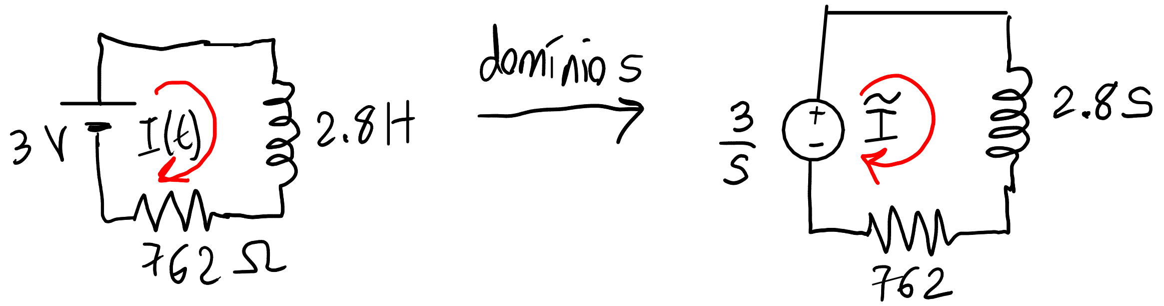
Exemplo 1. Uma bobina com $L = 2{,}8\,H$ e $R = 762\,\Omega$ liga-se a uma fonte de tensão, ideal, com $\mathcal{E} = 3\,V$. Determine a corrente na bobina 1 ms após ter sido ligada à fonte.

Resolução (SI)



$$\tilde{I} = \frac{3/s}{2.8s + 762} = \frac{\frac{3}{2.8}}{s\left(s + \frac{762}{2.8}\right)} = \frac{\frac{15}{14}}{s\left(s + \frac{1905}{7}\right)} = \frac{A}{s} + \frac{B}{s + \frac{1905}{7}}$$

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad \mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\frac{15}{14} = A\left(s + \frac{1905}{7}\right) + Bs$$

$$s = 0 \qquad A\left(\frac{1905}{7}\right) = \frac{15}{14} \qquad A = \frac{15}{2 \times 1905} = \frac{1}{254}$$

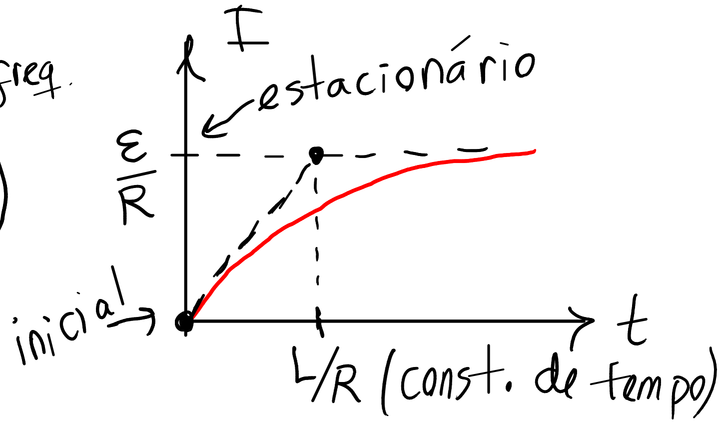
$$s = -\frac{1905}{7} \qquad B\left(-\frac{1905}{7}\right) = \frac{15}{14} \qquad B = -\frac{1}{254}$$

$$\Rightarrow I(t) = \frac{1}{254}\left(1 - e^{-\frac{1905}{7}t}\right) \qquad I(0.001) = \frac{1 - e^{-\frac{1.905}{7}}}{254}$$

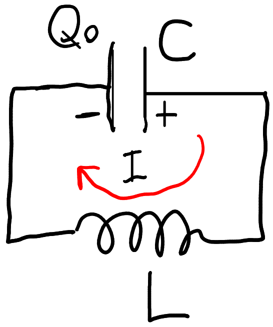
$$\boxed{I(0.001) = 0.938\ \text{mA}}$$

$$I(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-\overbrace{\tfrac{R}{L}}^{\text{freq.}}t}\right)$$

circuito LR

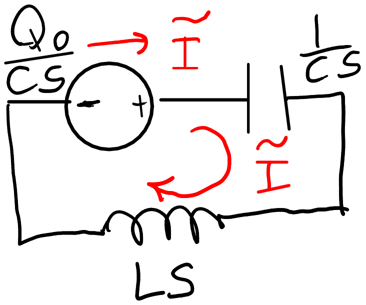


Exemplo 2. Um condensador, com carga inicial Q_0, liga-se a um indutor ideal. Determine a corrente no circuito, em função do tempo.



$$V_C(t) = \frac{Q}{C} = \frac{1}{C}\left(\int_0^t I\,dt - Q_0\right)$$

$$\tilde{V}_C = \mathcal{L}\{V_C\} = \frac{\tilde{I}}{Cs} - \frac{Q_0}{Cs} \leftarrow$$



$$\frac{\tilde{I}}{Cs} + \tilde{I}Ls - \frac{Q_0}{Cs} = 0$$

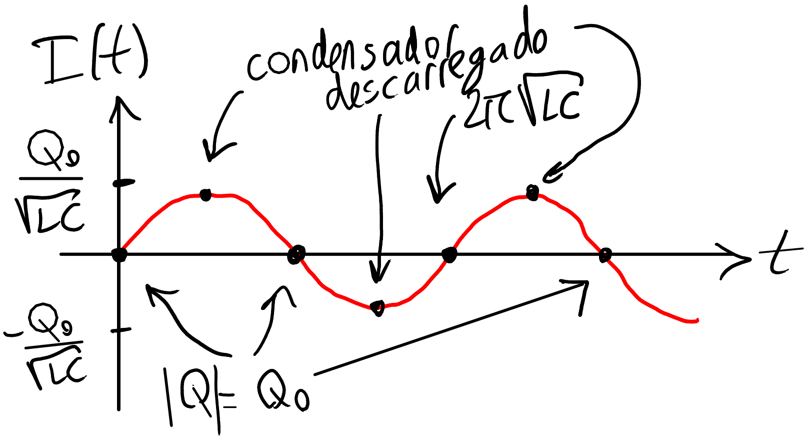
$$\tilde{I} = \frac{Q_0/Cs}{Ls + \frac{1}{Cs}} \leftarrow Z$$

$$I(t) = \mathcal{L}^{-1}\{\tilde{I}\} = \frac{Q_0}{\sqrt{LC}} \sin\left(\frac{t}{\sqrt{LC}}\right)$$

função seno com frequencia angular

$$\boxed{\omega = \frac{1}{\sqrt{LC}}}$$

freq. angular do circuito LC

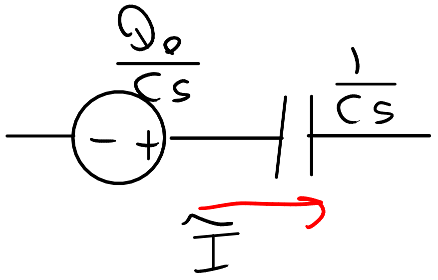


circuito oscilador

voltagem no indutor: $\tilde{V}_L = Z_L\tilde{I} = Ls\tilde{I}$

$$V_L(t) = \frac{Q_0}{C}\cos\left(\frac{t}{\sqrt{LC}}\right)$$

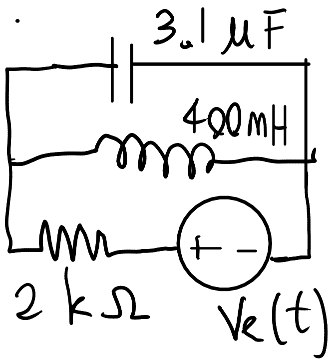
voltagem no condensador



$$\tilde{V}_C = \frac{\tilde{I}}{Cs} - \frac{Q_0}{Cs}$$

$$V_C(t) = -\frac{Q}{C}\cos\left(\frac{t}{\sqrt{LC}}\right) = -V_L(t)$$

Exemplo 3.



$$V_e(t) = 5\left(1 - e^{-600t}\right) \quad (SI)$$

em $t=0$, $Q_C = 0$, $I_L = 0$

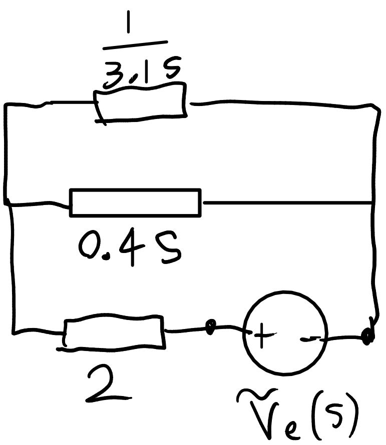
determine $I(t)$ na resistência.

Resolução. $\Delta V \to V$, $R \to k\Omega$, $C \to \mu F$ $\qquad \frac{1}{Cs} = Z$

$\Rightarrow Z \to k\Omega$, $I \to mA$, $s \to kHz$, $t \to ms$ $\qquad s = \frac{1}{CZ}$

$L \to H$ $\qquad Z = Ls$

$\qquad L = \frac{Z}{s}$



$t \to ms$: $V_e(t) = 5\left(1 - e^{-0.6t}\right)$

$$\tilde{V}_e(s) = \mathcal{L}\{V_e(t)\}$$

$$Z_t = 2 + \left(3.1s + \frac{1}{0.4s}\right)^{-1}$$

$$\tilde{I} = \frac{\tilde{V}_e}{Z_t} \to I(t) = \mathcal{L}^{-1}\{\tilde{I}\}$$

```
(%i1) Ve: 5*(1-exp(-0.6*t));
                                          - 0.6 t
(%o1)                        5 (1 - %e           )
(%i2) ve: laplace(Ve, t, s);
                                1      1
(%o2)                        5 (- - -------)
                                s    s + 0.6
(%i3) z: ratsimp(2+1/(3.1*s+1/0.4/s));
                                2
                            62 s  + 10 s + 50
(%o3)                       -----------------
                                   2
                               31 s  + 25
(%i4) i: ratsimp(ve/z);
                                    2
                               465 s  + 375
(%o4)                 --------------------------------
                           4        3        2
                      310 s  + 236 s  + 280 s  + 150 s
(%i5) I: ilt(i,s,t)$

(%i6) plot2d(I,[t,0,30],[ylabel,"I"])$
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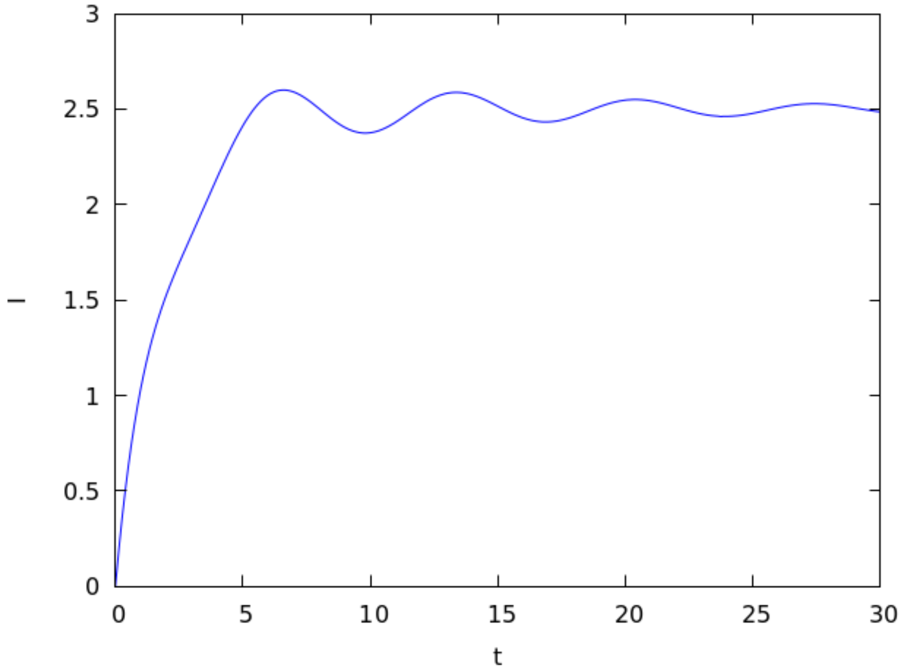
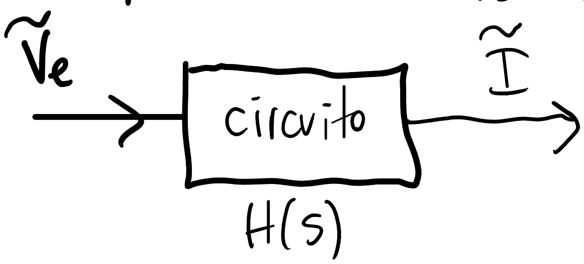


FUNÇÃO DE TRANSFERÊNCIA $(H(s))$



$$\boxed{\tilde{I} = H(s)\,\tilde{V}_e}$$

forma geral nos circuitos lineares

No exemplo 3, $H(s) = \frac{1}{Z_t} = \frac{31s^2+25}{62s^2+10s+50}$ $\left(\begin{matrix} s \text{ em} \\ \text{kHz} \end{matrix}\right)$