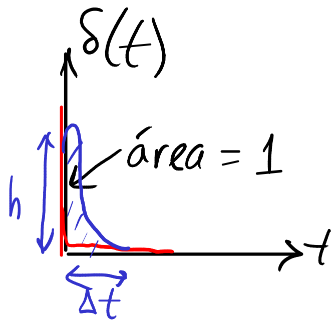
IMPULSO UNITÁRIO (delta de Dirac)

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \infty, & t = 0 \end{cases} \qquad \delta(t-4) = \begin{cases} 0, & t \neq 4 \\ \infty, & t = 4 \end{cases}$$

$$\int_0^\infty \delta(t) f(t)\, dt = f(0) \Rightarrow \int_0^\infty \delta(t)\, dt = 1$$



$$\mathcal{L}\{\delta(t)\} = \int_0^\infty \delta(t) e^{-st} dt = 1$$

$$\delta(t) \longrightarrow \boxed{\text{Sistema}} \xrightarrow{h(t)} \quad \overset{t \to s}{\rightsquigarrow} \quad 1 \longrightarrow \boxed{H(s)} \xrightarrow{H(s) = \mathcal{L}^{-1}\{h\}}$$

Maxima: $\delta(t) \rightarrow$ delta(t)

```
(%i1) laplace(delta(t),t,s);
(%o1)                          1
(%i2) ilt(1,s,t);
(%o2)                     ilt(1, s, t)
(%i3) f: (3*s^2+5*s-8)/(6*s^2+2*s-4)$

(%i4) ilt(f,s,t);
                             2
                          3 s  + 5 s - 8
(%o4)                 ilt(--------------, s, t)
                             2
                          6 s  + 2 s - 4
(%i5) partfrac(f,s);
                             1          1     1
(%o5)                   (- -------) + ----- + -
                          3 s - 2     s + 1   2
(%i6) ilt(%-1/2,s,t);
                                    2 t
                                    ---
                                     3
                            - t   %e
(%o6)                     %e    - -----
                                    3
(%i7) %+ delta(t)/2;
                                  2 t
                                  ---
                                   3
                     delta(t)   %e        - t
(%o7)                -------- - ----- + %e
                        2         3
```

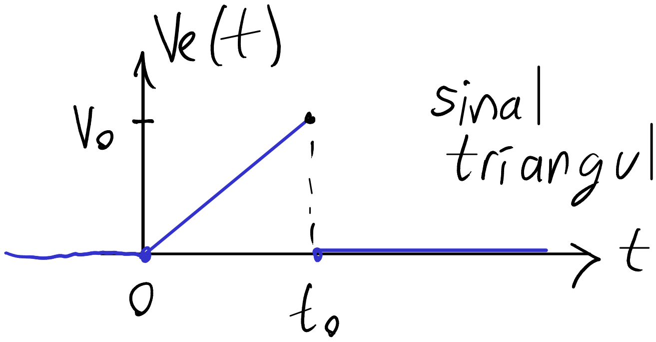
laplace calcula $\mathcal{L}\{\delta(t)\}$
mas ilt não calcula
$\mathcal{L}^{-1}\{1\}$

para obter $\mathcal{L}^{-1}\{\ \}$ duma função racional com polinómios do mesmo grau no numerador e denominador, podemos usar partfrac( )

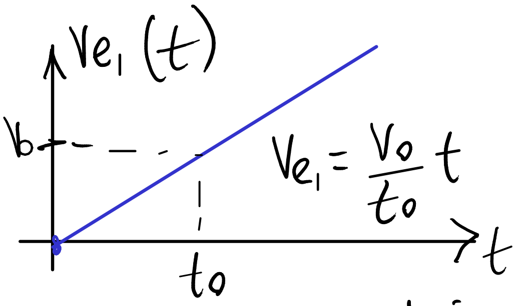
SINAIS PARCELARMENTE CONTÍNUOS

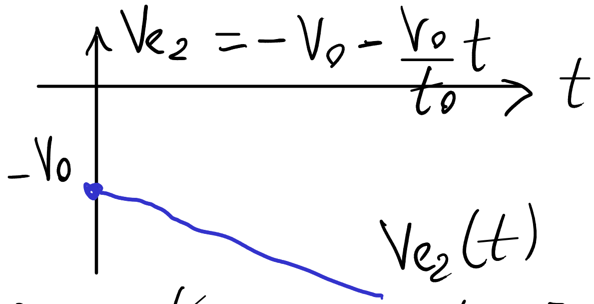
Exemplo 10.2 (livro).

filtro passa-alto



sinal triangular



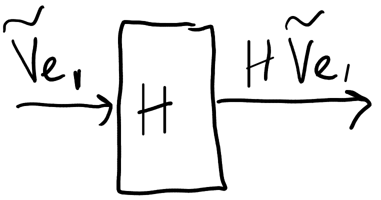


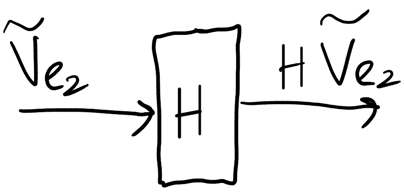
dois sinais contínuos em $t \in [0, \infty[$

$$V_e(t) = V_{e_1}(t) + u(t-t_0)\, V_{e_2}(t-t_0)$$

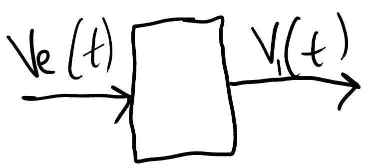
$$u(t-t_0) = \begin{cases} 0, & t < t_0 \\ 1, & t \geq t_0 \end{cases}$$

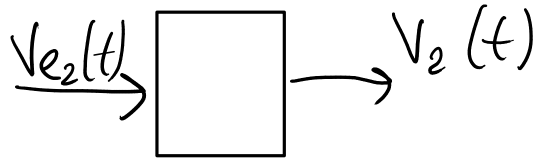
degrau unitário (função de Heaviside)

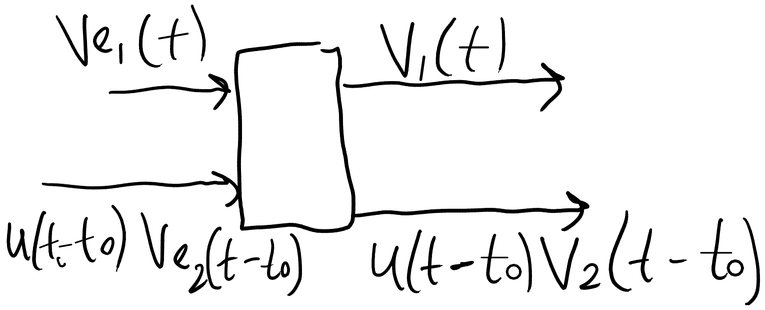




$s \to t$







$$\left.\right\} V(t) = V_1(t) + u(t-t_0)\,V_2(t-t_0)$$

$$Z = R + \frac{1}{Cs} \qquad \tilde{I} = \frac{\tilde{V}_e}{Z} = \frac{Cs}{RCs+1}\tilde{V}_e$$

$$\tilde{V} = R\tilde{I} = \frac{RCs}{RCs+1}\tilde{V}_e \Rightarrow \boxed{H(s) = \frac{RCs}{RCs+1}}$$

```
(%i8) H: RC*s/(RC*s+1);
                                  RC s
(%o8)                           --------
                                RC s + 1
(%i9) Ve1: V0*t/t0$
(%i10) ve1: laplace(Ve1,t,s);
                                  V0
(%o10)                           -----
                                  2
                                 s  t0
(%i11) v1: H*ve1;
                                 RC V0
(%o11)                      ---------------
                            s (RC s + 1) t0
(%i12) V1: ilt(v1,s,t);
                                       - t/RC
                         RC V0    RC V0 %e
(%o12)                   -----  - --------------
                          t0            t0
(%i13) Ve2: -V0-V0*t/t0$
(%i14) ve2: laplace(Ve2,t,s);
                                 V0       V0
(%o14)                       (- -----) - --
                                 2        s
                                s  t0
(%i15) v2: H*ve2;
                                   V0       V0
                         RC s ((- -----) - --)
                                   2        s
                                  s  t0
(%o15)                   ---------------------
                               RC s + 1
(%i16) V2: ilt(v2,s,t);
                      - t/RC                 2
                    %e        (RC V0 t0 - RC   V0)     RC V0
(%o16)            (- ------------------------------) - -----
                              RC t0                      t0
```

```
(%i17) V: V1 + unit_step(t-t0)*subst(t=t-t0,V2);
                                                       t - t0
                                                     - ------
                                              2          RC
                             (RC V0 t0 - RC   V0) %e              RC V0
(%o17) unit_step(t - t0) ((- ------------------------------) - -----)
                                        RC t0                     t0
                                                                  - t/RC
                                                         RC V0 %e          RC V0
                                                     - -------------- + -----
                                                             t0           t0
```

(Capítulo 12. Não entra no exame)

4 equações de Maxwell

1. Lei de Gauss $\oiint_S (\vec{E} \cdot \hat{n})\, dA = 4\pi k\, q_{int}$

2. $\oint_C \vec{E} \cdot d\vec{r} = -\frac{\partial}{\partial t} \iint_S (\vec{B} \cdot \hat{n})\, dA$ (Lei de Faraday)

3. $\oiint_S (\vec{B} \cdot \hat{n})\, dA = 0$ (não há monopolos mag.)

4. lei de ampére: $\oint_C \vec{B} \cdot d\vec{r} = 4\pi k_m I_{int} + \underbrace{}_{\text{termo de Maxwell } \left(\frac{\partial \vec{E}}{\partial t}\right)}$

$q = 0$, $I = 0$ → podem existir $\vec{B}$ e $\vec{E} \neq \vec{0}$ (onda eletromagnética)

com velocidade $v = \sqrt{\frac{k}{k_m}}$

$$v = \sqrt{\frac{9 \times 10^9\ N \cdot m^2/C^2}{10^{-7}\ \ldots}} = 3 \times 10^8\ \frac{m}{s}$$ (= velocidade da luz!)