# Sumário de Mecânica Quântica

### Espaços vetoriais com coeficientes complexos

$$|\Psi\rangle + (|\Phi\rangle + |\Upsilon\rangle) = (|\Psi\rangle + |\Phi\rangle) + |\Upsilon\rangle \qquad |\Psi\rangle + |\Phi\rangle = |\Phi\rangle + |\Psi\rangle$$

$$|\Psi\rangle + |0\rangle = |\Psi\rangle \qquad |\Psi\rangle + |-\Psi\rangle = |0\rangle \qquad z(|\Psi\rangle + |\Phi\rangle) = z|\Psi\rangle + z|\Phi\rangle$$

$$z(w|\Psi\rangle) = zw|\Psi\rangle \qquad (z+w)|\Psi\rangle = z|\Psi\rangle + w|\Phi\rangle \qquad \langle\Psi|\Phi\rangle = \langle\Phi|\Psi\rangle^*$$

$$\langle\Psi|(|\Phi\rangle + |\Upsilon\rangle) = \langle\Psi|\Phi\rangle + \langle\Psi|\Upsilon\rangle \qquad z\langle\Psi|\Phi\rangle = \langle\Psi|(z|\Phi\rangle) \qquad \langle\Psi|\Psi\rangle = ||\Psi\rangle|^2 \geq 0$$

### Bases e componentes

$$\langle \mathrm{e}_j|\mathrm{e}_k\rangle = \delta_{jk} \qquad |\Psi\rangle = \sum_{j=1}^{n} \Psi_j|\mathrm{e}_j\rangle \qquad \Psi_j = \langle \mathrm{e}_j|\Psi\rangle \qquad \langle\Psi| = \sum_{j=1}^{n} \Psi_j^*\langle \mathrm{e}_j|$$

$$\langle\Psi|\Phi\rangle = \sum_{j=1}^{n} \Psi_j^*\Phi_j \qquad \langle\Psi|\Psi\rangle = \sum_{j=1}^{n} |\Psi_j|^2 = ||\Psi\rangle|^2 \quad \text{(igual a 1 nas próximas linhas)}$$

$$\hat{\Omega}|\Psi\rangle = \sum_{j=1}^{n}\sum_{k=1}^{n} \Omega_{jk}\Psi_k|\mathrm{e}_j\rangle \qquad \Omega_{jk} = \langle \mathrm{e}_j|\hat{\Omega}|\mathrm{e}_k\rangle \qquad \langle\hat{\Omega}\rangle = \langle\Psi|\hat{\Omega}|\Psi\rangle = \sum_{j=1}^{n}\sum_{k=1}^{n} \Omega_{jk}\Psi_j^*\Psi_k$$

### Representação matricial

$$|\Psi\rangle = \begin{bmatrix}\Psi_1\\ \Psi_2\\ \vdots\\ \Psi_n\end{bmatrix} \qquad \langle\Psi| = \begin{bmatrix}\Psi_1^* & \Psi_2^* & \dots & \Psi_n^*\end{bmatrix} \qquad \hat{\Omega} = \begin{bmatrix}\Omega_{11} & \Omega_{12} & \dots & \Omega_{1n}\\ \Omega_{21} & \Omega_{22} & \dots & \Omega_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \Omega_{n1} & \Omega_{n2} & \dots & \Omega_{nn}\end{bmatrix}$$

### Valores e vetores próprios

$$\hat{\Omega}|\lambda\rangle = \lambda\,|\lambda\rangle \qquad \begin{vmatrix}\Omega_{11}-\lambda & \Omega_{12} & \dots & \Omega_{1n}\\ \Omega_{21} & \Omega_{22}-\lambda & \dots & \Omega_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \Omega_{n1} & \Omega_{n2} & \dots & \Omega_{nn}-\lambda\end{vmatrix} = 0$$

### Observáveis (operadores hermíticos)

$\Omega_{jk} = \Omega_{kj}^*$ Valores próprios reais: $\lambda = \lambda^*$ Base ortonormal: $\langle\lambda_j|\lambda_k\rangle = \delta_{jk}$

Possíveis valores medidos: $\lambda_k$
Probabilidade de medir $\lambda_k$: $\mathrm{P}_k = \langle\Psi|\lambda_k\rangle\langle\lambda_k|\Psi\rangle$
Estado após medir $\lambda_k$: $|\lambda_k\rangle$

### Comutadores

$$[\hat{\Omega},\hat{\Lambda}] = \hat{\Omega}\hat{\Lambda} - \hat{\Lambda}\hat{\Omega} \qquad [\hat{x},\hat{p}] = \mathrm{i}\,\hbar \qquad \hbar = 1.055\times 10^{-34}\ \frac{\mathrm{kg\cdot m^2}}{\mathrm{s}}$$

$$[\hat{\Omega},\hat{\Lambda}] = -[\hat{\Lambda},\hat{\Omega}] \qquad [\hat{\Omega},\hat{\Omega}] = 0 \qquad \hat{\Omega}\hat{\Lambda} = \hat{\Lambda}\hat{\Omega} + [\hat{\Omega},\hat{\Lambda}]$$

**Matrizes de Pauli**

$$\hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad \hat{\sigma}_y = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix} \qquad \hat{\sigma}_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\hat{\sigma}_j \hat{\sigma}_k = -\hat{\sigma}_k \hat{\sigma}_j \qquad \hat{\sigma}_x \hat{\sigma}_y = \mathrm{i}\hat{\sigma}_z \qquad \hat{\sigma}_y \hat{\sigma}_z = \mathrm{i}\hat{\sigma}_x \qquad \hat{\sigma}_z \hat{\sigma}_x = \mathrm{i}\hat{\sigma}_y$$

**Equação de Schrödinger**

$$\mathrm{i}\hbar \frac{\mathrm{d}|\Psi\rangle}{\mathrm{d}t} = \hat{\mathrm{H}}|\Psi\rangle \qquad \hat{\mathrm{H}}|\mathrm{E}_j\rangle = E_j|\mathrm{E}_j\rangle \qquad |\Psi(0)\rangle = \sum_{j=1}^{n} \Psi_j |\mathrm{E}_j\rangle$$

**Oscilador harmónico**

$$\hat{\mathrm{H}} = \frac{1}{2m}\hat{\mathrm{p}}^2 + \frac{k}{2}\hat{\mathrm{x}}^2 \qquad \omega = \sqrt{\frac{k}{m}} \qquad E_j = \hbar\omega\left(j + \frac{1}{2}\right) \quad (j = 0, 1, 2, \ldots)$$

**Combinação de dois sistemas**

$$|\Psi, \Phi\rangle = |\Psi\rangle \otimes |\Phi\rangle \qquad \langle \mathrm{e}_j, \mathrm{e}_k | \mathrm{e}_r, \mathrm{e}_s\rangle = \delta_{jr}\delta_{ks} \qquad |\Psi, \Phi\rangle = \sum_{j=1}^{n}\sum_{r=1}^{m} \Psi_j \Phi_r |\mathrm{e}_j, \mathrm{e}_r\rangle$$

$$\left(\hat{\Omega} \otimes \hat{\Lambda}\right)|\Psi, \Phi\rangle = \hat{\Omega}|\Psi\rangle \otimes \hat{\Lambda}|\Phi\rangle = \sum_{j=1}^{n}\sum_{r=1}^{m} \Omega_{jk}\Lambda_{rs}\Psi_k\Phi_s |\mathrm{e}_j, \mathrm{e}_r\rangle$$

Estados entrelaçados: $|\Upsilon\rangle = \sum_{j=1}^{n}\sum_{r=1}^{m} \Upsilon_{jr} |\mathrm{e}_j, \mathrm{e}_r\rangle$

$$\left(\hat{\Omega} \otimes \hat{\Lambda}\right)|\Upsilon\rangle = \sum_{j=1}^{n}\sum_{r=1}^{m} \Omega_{jk}\Lambda_{rs}\Upsilon_{ks} |\mathrm{e}_j, \mathrm{e}_r\rangle$$