A NEW PROTOTYPE TO SEPARATE SOLID PARTICLES BY CENTRIFUGAL AND MAGNETIC FORCES: FURTHER DEVELOPMENTS

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ABSTRACT

This work presents the advances made on a new separator where the particles are submitted to the simultaneous action of two central force fields: centrifugal and magnetic. The magnetic field generator presented in (1) was modified and is now based on a superconductor (2). Also the approximations made in the initial analysis (1) were more adjusted to reality. New mathematical equations were derived so that the mathematical model developed may be more adjusted to the present reality of the separator. The limiting conditions of the manipulated variables are presented for each phase of the separation. A complete simulation of a typical separation will be presented as well as some design parameters.

Typical Open-Gradient magnetic separators and High-Gradient magnetic separators only deflect or collect (respectively) the magnetic particles (3), (4), (5) without having the ability to separate them by classes as accurate as one wishes. The separator in consideration may achieve that kind of degree of separation due to its peculiar geometry.

PROTOTYPE DESIGN

The Fig. 1, shows the general configuration of the separator. Being composed by a rotating conical-trunk where the feed leaves the particles, and by a fixed central superconducting magnet surrounded by a collecting grid, the main ability of the separator is to exert a combination of forces on the particles such that the non-magnetic ones rise on the surface, and the magnetic ones descend on it until a critical point is reached where they will depart towards the centre, being collected there by a specially designed grid.

The conical configuration is adopted because with this configuration the particles will suffer a higher magnetic force as they are descending the separator's surface in a natural way - without having to increase the value of the current intensity at the superconducting magnet: the magnetic gradient needed for the separation is naturally achieved. This results in a process where the particles with higher magnetic susceptibility will depart sooner reaching the collecting grid at a higher level than the following ones. This way, by designing the grid properly, we may collect the magnetic particles by classes*.

The main difference of this configuration relating to the one presented in (1) is the substitution of the electromagnet generator of the magnetic field by a wire superconductor magnet, because this allows a better separation and manipulation.

MATHEMATICAL MODEL (2), (6), (7)

In this section the mathematical model presented in (1) is extensively revised as applied to the new prototype configuration, the limiting conditions of the manipulated variables and intrinsic characteristics of the particles that must be maintained for the occurrence of the separation are presented and new approximations, more related to the practical reality, are considered.

* Classes are defined as a group of magnetic particles with magnetic permeability between two limits which (theoretically) can be as tight as one wishes.

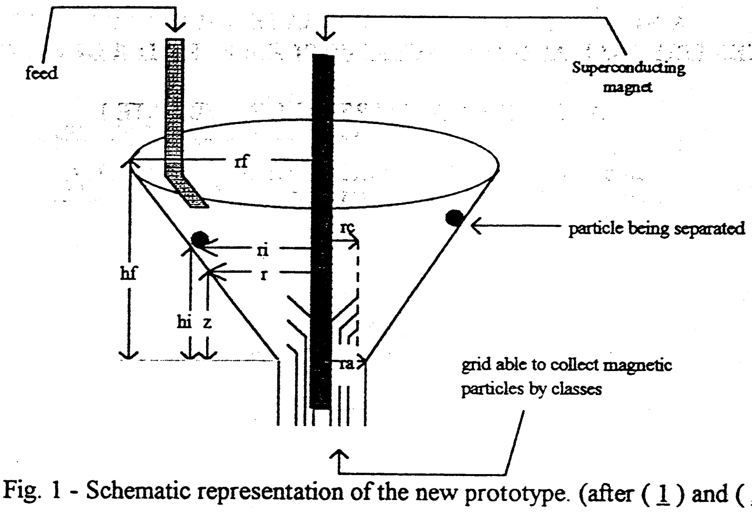

Fig. 1 - Schematic representation of the new prototype. (after (1) and (2))

APPROXIMATIONS CONSIDERED AND T_{CG}'S

The following assumptions were considered in the development of the mathematical model:
- the interparticle forces are negligible;
- the particles are spherical or quasi-spherical;
- the particles are assumed to have an initial null (or low) velocity on the tangential direction, and to acquire immediately the radial velocity of the conical-trunk or having it as an initial value;
- the friction coefficient is always constant, even at the start of the particle's movement;
- the friction coefficient in the radial direction is such that the particles "glue" to the separator's surface in that direction (but move on the other ones), so that while on the surface the centrifugal force exerts its action on the particle fully;
- the very weakly magnetic particles which go upwards on the separator's surface (because of their very low magnetic susceptibility) are analysed as normal non-magnetic ones;
- the drag forces acting on the non-magnetic particles are negligible (because the movement of the air is also upwards though without as much velocity as the particles);
- the movement of the air due to the rotation is neglected;
- the superconducting magnet behaves like a wire with infinite length;
- the density of the fluid medium (air) is neglected in relation to the density of the particle, and the centrifugal and magnetic forces are considered much greater than the drag forces. This prevents the possibility of a centrifugal classification by densities effect (8);

Any deviations to these approximations are accounted by the so-called theoretical coefficient global 1 - T_{CG1} (" Coeficente global teórico 1) , for the non-magnetic particles, and theoretical coefficient global 2 - T_{CG2} (" Coeficente global teórico 2) , for the magnetic particles. They are defined by the following equations (1),

$r_R = T_{CG1} \cdot r_i$ - for non-magnetic particles (1)

$r_R = T_{CG2} \cdot r_i$ - for magnetic particles (2)

The physical interpretation of these two coefficients is the following one : the real initial radius of the real separating system is r_i , but for the idealised mathematical system to adjust the reality of deviations we must think of an idealised initial radius r_R (*R* stands for the reality of deviations though this is a non-real radius) which is related to the real radius r_i by the coefficients T_{CG1} and T_{CG2} , but when considered to be the initial radius on the mathematical model proposed, manages to get the mathematical model to work well. So the T_{CG}'s coefficients are a measure of the deviation of the idealised system from reality.

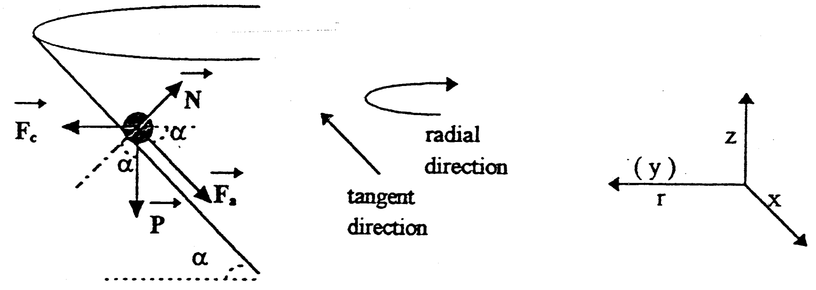

Fig. 2 - Schematic representation of the force-balance in a non-magnetic particle.

The non-magnetic particles will suffer the action of several forces as represented in Fig. 2. These forces are:

$F_C = Vol_p \cdot (\rho_p - \rho_f) \cdot \omega^2 \cdot r$ (8) $\cong^* m \cdot \omega^2 \cdot r$ (3)

$P = Vol_p \cdot (\rho_p - \rho_f) \cdot g$ (7) $\cong^* m \cdot g$ (4)

$F_a = \mu \cdot N$ (7) (5)

So, balancing these forces in the 3 directions, and bearing in mind that

$t = 0 \Rightarrow r = T_{CG1} \cdot r_i ,\ V_{rt} = 0$ (7)

we get the following expressions:

$$r = \left(\frac{T_{CG1} \cdot r_i}{2} + \frac{b}{2 \cdot a}\right) \cdot \left(e^{\sqrt{a} \cdot t} + e^{-\sqrt{a} \cdot t}\right) - \frac{b}{a} \quad (8)$$

$$z = r \cdot tg\, \alpha \quad (9)$$

tangential direction,

$$V_{rt} = \frac{\sqrt{a}}{\cos\alpha} \cdot \left(\frac{T_{CG1} \cdot r_i}{2} + \frac{b}{2 \cdot a}\right) \cdot \left(e^{\sqrt{a} \cdot t} - e^{-\sqrt{a} \cdot t}\right) \quad (10)$$

radial direction (note: *radial* not *r*),

$$V_\theta = \omega \cdot \left(\frac{T_{CG1} \cdot r_i}{2} + \frac{b}{2 \cdot a}\right) \cdot \left(e^{\sqrt{a} \cdot t} + e^{-\sqrt{a} \cdot t}\right) - \omega \cdot \frac{b}{a} \quad (11)$$

where on the equations (8)-(11),

$$\frac{b}{a} = -\frac{(tg\alpha + \mu) \cdot g}{(1 - \mu \cdot tg\alpha) \cdot \omega^2} \quad (12) \quad , \quad \sqrt{a} = \cos\alpha \cdot \omega \cdot \sqrt{1 - \mu \cdot tg\alpha} \quad (13)$$

Limiting conditions

Bearing in mind that for the separation of non-magnetic particles in a direction pointing upwards to be possible $F_{Rt} > 0$, we may deduct that we must have the following conditions:

Pair number one

$$\begin{cases} 0 < \alpha < arctg\left(\dfrac{-g \cdot \mu + \omega^2 \cdot T_{CG} \cdot ri}{\mu \cdot \omega^2 \cdot T_{CG} \cdot ri + g}\right) & (14) \\ \omega > \sqrt{\dfrac{g \cdot \mu}{T_{CG} \cdot ri}} & (15) \end{cases}$$

OR

Pair number two

$$\begin{cases} 0 < \alpha < arctg\left(\dfrac{1}{\mu}\right) & (16) \\ \omega > \sqrt{\dfrac{(tg\alpha + \mu) \cdot g}{(1 - \mu \cdot tg\alpha) \cdot T_{CG} \cdot ri}} & (17) \end{cases}$$

The difference between the two pairs resides in (17) representing a higher limit for ω than (15), and (14) representing a more limiting expression for α than (16). So , in cases where

* $\rho_p >> \rho_f$

a less limiting condition to ω is needed we choose the pair number one as the right one, "sacrificing" α this way, and in cases where a less limiting condition for α is needed we choose the pair number two, "sacrificing" ω this way.
The condition limiting the radius of the non-magnetic particles (in order to avoid collisions between them) may be found if we analyse what happens from turn to turn - how much "ground" is won in a turn -, and it may be proven that the turn which wins less "ground" is the first one, and so,

$$r_{mp} \leq \frac{\Delta S_{minor}}{2} = \frac{\left(\frac{T_{CG} \cdot r_i}{2 \cdot \cos\alpha} + \frac{b}{2 \cdot a \cdot \cos\alpha}\right) \cdot \left(e^{\sqrt{a} \cdot T} + e^{-\sqrt{a} \cdot T} - 2\right)}{2} \qquad (18)$$

MAGNETIC PARTICLES

The movement of the magnetic particles must be divided in three distinctive parts:
a) particle moving supported by the conical-trunk surface ($r_{lv} \leq r \leq T_{CG2} \cdot r_i$)
b) moment when particle departs from the conical-trunk surface ($r = r_{lv}$)
c) particle moving freely through the fluid towards the centre ($r \leq r_{lv}$)

a) Particle moving supported by the conical-trunk surface ($r_{lv} \leq r \leq T_{CG2} \cdot r_i$)

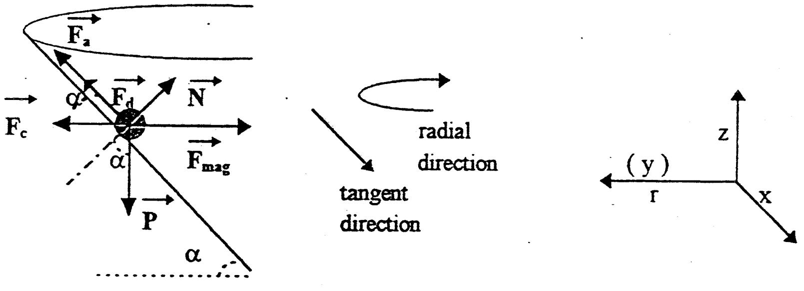

Fig. 3 - Schematic representation of the force-balance in a magnetic particle.

The non-magnetic particles will suffer the action of several forces as represented in Fig. 3. These forces are:

$$F_C = Vol_p \cdot (\rho_p - \rho_f) \cdot \omega^2 \cdot r \ (8) \cong m \cdot \omega^2 \cdot r \qquad (3)$$

$$P = Vol_p \cdot (\rho_p - \rho_f) \cdot g \ (7) \cong m \cdot g \qquad (4)$$

$$F_a = \mu \cdot N \quad (7) \qquad (5)$$

$$F_d = C_d \cdot A \cdot \frac{\rho_f \cdot V_p^2}{2} \ (9) = m \cdot C_d \cdot \frac{A}{Vol_p} \cdot \frac{\rho_f}{\rho_p} \cdot \frac{V_p^2}{2} \qquad (19)$$

As to the magnetic force we may show (1), (3), (4), (5), that if the particle is not very large, the susceptibility of the fluid is negligible and the gradient of the field is only present in the r direction, then, considering (only for the magnetic force) as the positive direction the one pointing towards the centre:

$$F_{mag} = -\frac{m \cdot \chi}{\mu_0} \cdot B \cdot \frac{\partial B}{\partial r} \qquad (20)$$

For the variation of B with r, we will admit as an approach to the field generated by the superconducting wire (7),

$$B = \frac{\mu_0 \cdot I}{2 \cdot \pi \cdot r} = \frac{a}{r} \quad with \quad a = \frac{\mu_0 \cdot I}{2 \cdot \pi} \qquad (21)$$

and so,

$$F_{mag} = \frac{m \cdot a^2 \chi}{\mu_0 \cdot r^3} \quad (22)$$

So, balancing these forces in the 3 directions, and keeping in mind that

$$t = 0 \Rightarrow r = T_{CG2} \cdot r_i,\ V_t = 0 \quad (23)$$

we get the following expressions:

tangential direction,

$$V_t(r) = \sqrt{A + B + C + D} \quad (24)$$

where,

$$A = \frac{q_2}{T_{CG_2}^2 \cdot r_i^2} \cdot e^{-q_1\left(r - T_{CG_2} \cdot r_i\right)} - \frac{q_2}{r^2} + \frac{q_2 \cdot q_1}{T_{CG_2} \cdot r_I} \cdot e^{-q_1\left(r - T_{CG_2} \cdot r_i\right)} - \frac{q_2 \cdot q_1}{r} \quad (25)$$

$$B = -\frac{2}{q_1} \cdot \left(q_4 - \frac{q_3}{q_1}\right) \cdot e^{-q_1\left(r - T_{CG_2} \cdot r_i\right)} + \frac{2}{q_1} \cdot \left(q_4 - \frac{q_3}{q_1}\right) \quad (26)$$

$$C = q_2 \cdot q_1^2 \cdot e^{-q_1 \cdot r} \cdot \int_{T_{CG2} \cdot r_i}^{r} \frac{e^{q_1 \cdot r}}{r} dr \quad (27)$$

$$D = -\frac{2 \cdot q_3 \cdot T_{CG_2} \cdot r_i}{q_1} \cdot e^{-q_1\left(r - T_{CG_2} \cdot r_i\right)} + \frac{2 \cdot q_3 \cdot r}{q_1} \quad (28)$$

and

$$q_1 = -\frac{3}{4} \cdot \frac{C_d}{r_{part} \cdot \cos\alpha},\ q_2 = -\frac{a^2 \cdot \chi}{\mu_0} \cdot (1 + \mu \cdot tg\alpha),\ q_3 = \omega^2 \cdot (1 + \mu \cdot tg\alpha),\ q_4 = -g \cdot (tg\alpha - \mu) \quad (29)$$

the integral $\int_{T_{CG2} \cdot r_i}^{r} \frac{e^{q_1 \cdot r}}{r} dr$ may be solved numerically or following the approximation (6),

$$\int \frac{e^{q_1 \cdot r}}{r} dr = \ln(r) + \sum_{i=1}^{n} \frac{(q_1 \cdot r)^i}{i! \, i} \quad (30)$$

with an error, $\int_0^{q_1 \cdot r} \frac{e^t}{n!} \cdot (q_1 \cdot r - t)^n dt$ (31)

Now, with the expression (24) in mind we may see that,

$$t = \int_r^{T_{CG2} \cdot r_i} \frac{dr}{\cos\alpha \cdot V_t(r)} \quad (32)$$

and so after calculating $V_t(r)$ we may calculate $r = f(t)$ numerically, and then substitute this value in (27) in order to get $V_t(t)$.

We may also recall,

$$z = r \cdot tg\,\alpha \quad (9)$$

which is valid for this phase of the movement.

Radial direction,

$$V_\theta(t) = \omega \cdot r(t) \quad (33)$$

Limiting conditions

The limiting conditions may be derived from the main condition $F_{Rt} > 0$. Solving this condition we may prove that we must have the following limiting conditions:

$$
(SMA1A)\begin{cases}
\omega^2 < \dfrac{E+F+G+H}{I} & (34)\\[2ex]
\dfrac{a^2\cdot\chi}{g\cdot\mu_0} > \dfrac{\dfrac{tg\alpha-\mu}{1+\mu\cdot tg\alpha}}{\dfrac{T_{CG2}\cdot r_i\cdot e^{q_1\cdot T_{CG2}\cdot r_i\cdot(\lambda-1)}\cdot(E+F+H)}{q_2}} \quad if \quad \dfrac{T_{CG2}\cdot r_i\cdot e^{q_1\cdot T_{CG2}\cdot r_i\cdot(\lambda-1)}\cdot(E+F+H)}{q_2}<0 & (35)\\[2ex]
\dfrac{a^2\cdot\chi}{g\cdot\mu_0} < \dfrac{\dfrac{tg\alpha-\mu}{1+\mu\cdot tg\alpha}}{\dfrac{T_{CG2}\cdot r_i\cdot e^{q_1\cdot T_{CG2}\cdot r_i\cdot(\lambda-1)}\cdot(E+F+H)}{q_2}} \quad if \quad \dfrac{T_{CG2}\cdot r_i\cdot e^{q_1\cdot T_{CG2}\cdot r_i\cdot(\lambda-1)}\cdot(E+F+H)}{q_2}>0, tg\alpha>\mu & (36)
\end{cases}
$$

where

$$
E=-\frac{q_2\cdot q_1}{2\cdot T_{CG_2}^3\cdot r_I^3}\cdot\left(\frac{1}{\lambda^2}-e^{-q_1\cdot T_{CG_2}\cdot r_i\cdot(\lambda-1)}\right)\ (37)\ ,F=-\frac{q_2\cdot q_1^2}{2\cdot T_{CG_2}^2\cdot r_I^2}\cdot\left(\frac{1}{\lambda}-e^{-q_1\cdot T_{CG_2}\cdot r_i\cdot(\lambda-1)}\right)\ (38)\ ,
$$

$$
G=-\frac{q_4}{T_{CG_2}\cdot r_i}\cdot e^{-q_1\cdot T_{CG_2}\cdot r_i\cdot(\lambda-1)}\ (39),\ H=\frac{q_2\cdot q_1^3}{2\cdot T_{CG_2}\cdot r_i}\cdot e^{-q_1\cdot T_{CG_2}\cdot r_i\cdot\lambda}\cdot\int_1^\lambda\frac{e^{q_1\cdot T_{CG_2}\cdot r_i\cdot\lambda}}{\lambda}d\lambda-\frac{q_2}{\lambda^3\cdot T_{CG_2}^4\cdot r_i^4}\ (40),
$$

$$
I=(\cos\alpha+\mu\cdot sen\alpha)\cdot\left(e^{-q_1\cdot T_{CG_2}\cdot r_i\cdot(\lambda-1)}-\frac{e^{-q_1\cdot T_{CG_2}\cdot r_i\cdot(\lambda-1)}}{q_1\cdot T_{CG_2}\cdot r_i}+\frac{1}{q_1\cdot T_{CG_2}\cdot r_i}\right)\ (41),\ \lambda=\frac{r}{T_{CG_2}\cdot r_i}\ (42)
$$

OR

$$
(SMA1B)\begin{cases}
\omega^2 < \dfrac{E+F+G+H}{I} & (34)\\
E+F+G+H>0 \text{ and knowing the other parameters } (\chi,\ a^2) \text{ we may implicitly} & \\
\text{obtain a condition for } \alpha \text{ or } r_{part}. & (43)
\end{cases}
$$

OR

$$
(SMA2)\begin{cases}
E+F+G+H-\dfrac{q_3}{(\cos\alpha+\mu\cdot sen\alpha)}\cdot I>0 \text{ and knowing the other parameters } (\chi,\ a^2,\omega) & \\
\text{we may implicitly obtain a condition for } \alpha \text{ or } r_{part}. & (44)
\end{cases}
$$

OR

$$
(SMA3)\begin{cases}
\dfrac{a^2\cdot\chi}{\mu_0} < \dfrac{\dfrac{q_3}{\cos\alpha+\mu\cdot sen\alpha}\cdot I-G}{(1+\mu\cdot tg\alpha)\cdot\dfrac{(E+F+H)}{-q_2}} \quad if \quad \dfrac{\overbrace{(E+F+H)}^{f(\alpha,r_{part})}}{-q_2}<0\wedge\begin{cases}\omega^2<\dfrac{\dfrac{g\cdot(tg\alpha-\mu)}{1+\mu\cdot tg\alpha}}{I1}\ if\ I1>0, tg\alpha>\mu & (45)\\[2ex] \omega^2>\dfrac{\dfrac{g\cdot(tg\alpha-\mu)}{1+\mu\cdot tg\alpha}}{I1}\ if\ I1<0 & (46)\end{cases}\\[4ex]
\dfrac{a^2\cdot\chi}{\mu_0} > \dfrac{\dfrac{q_3}{\cos\alpha+\mu\cdot sen\alpha}\cdot I-G}{(1+\mu\cdot tg\alpha)\cdot\dfrac{(E+F+H)}{-q_2}} \quad if \quad \dfrac{\overbrace{(E+F+H)}^{f(\alpha,r_{part})}}{-q_2}>0 & (47)
\end{cases}
$$

$$
\text{where } I1=\frac{e^{q_1\cdot T_{CG_2}\cdot r_i\cdot(\lambda-1)}-1+q_1\cdot T_{CG_2}\cdot r_i}{q_1} \qquad (48)
$$

To understand the reason of several systems of conditions instead of only one, we must realise that they represent the same general limiting condition but that allow each manipulated variable to have a value more or less limited at the expenses of limiting less or

more, respectively, the other manipulated variables. This results in a flexible system of limiting conditions. For example (SMA1A) represents a more limited value for ω^2 and a less limited value for $a^2\chi$ but the opposite happens in (SMA3) where $a^2\chi$ is more limited and ω^2 less limited than in SMA1A. The limiting values of $a^2\chi$ and α, r_{part} are presented together because they have the same limiting condition, and in order to determine one parameter we must know the other one (except in the cases where we have a limiting condition for $f(\alpha, r_{part})$ and another limiting condition for α alone).
On the other hand, we need that the friction acting in the radial direction is enough to avoid the effects of the drag force on the particles' radial movement. In order fulfil this condition,

$$F_a \geq F_d \Rightarrow \mu \geq \frac{\frac{3}{8}\cdot\frac{C_d}{r_{part}}\cdot\frac{\rho_f}{\rho_p}\cdot\omega^2\cdot r^2}{w^2\cdot r\cdot sen\alpha + g\cdot\cos\alpha - \frac{a^2\cdot\chi}{\mu_0\cdot r^3}\cdot sen\alpha} \quad (49)$$

b) Transition ($r = r_{lv}$)

To calculate r_{lv}, the point where the particle departs from the separator's surface heading towards the centre, we must set N=0, and so,

$$\omega^2\cdot r_{lv}^4 + \cot g\alpha\cdot g\cdot r_{lv}^3 - \frac{a^2\cdot\chi}{\mu_0} = 0 \quad (50)$$

from where we calculate the value of r_{lv}, as a function of ω, a^2, χ and α.
Now, if we solve the equation N=0 in order to χ and realise that the particles which arrive to r_a without jumping towards the centre, will fall on the base collector, we may determine the critical magnetic susceptibility below which the particles will not depart,

$$\chi < \frac{\mu_0\cdot\omega^2\cdot r_a^4}{a^2} + \frac{\mu_0\cdot\cot g\alpha\cdot g\cdot r_a^3}{a^2} \quad (51)$$

Solving N=0 in order to the other manipulated variables allow us to control exactly which class of particles departs at a determined radius r_{lv} chosen by us.

c) Particle moving freely through the fluid towards the centre ($r \leq r_{lv}$)

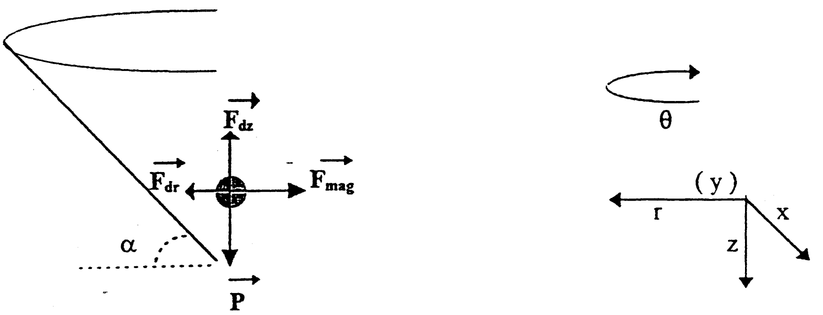

Fig. 4 - Schematic representation of the force-balance in magnetic particles "flying".
The non-magnetic particles will suffer the action of several forces as represented in Fig. 4. These forces are:

$$P = Vol_p\cdot(\rho_p-\rho_f)\cdot g\ (7) \cong m\cdot g \quad (4)$$

$$F_d = C_d\cdot A\cdot\frac{\rho_f\cdot V_p^2}{2}\ (9) = m\cdot C_d\cdot\frac{A}{Vol_p}\cdot\frac{\rho_f}{\rho_p}\cdot\frac{V_p^2}{2} \quad (19)$$

$$F_{mag} = \frac{m \cdot a^2 \chi}{\mu_0 \cdot r^3} \tag{22}$$

and equating the forces in the three directions, keeping in mind that

$$t = t_{lv} \Rightarrow r = r_{lv} \Rightarrow V_r = V_{rlv},\ V_z = V_{zlv},\ V_\theta = V_{\theta lv} = \omega \cdot r_{lv} \tag{52}$$

we may derive the following expressions,

r direction (neglecting the contribution of $V_{\theta lv}$ in this direction):

$$V_r(r) = \sqrt{J + K + L + M} \tag{53}$$

where

$$J = \frac{K_b}{r_{lv}^2} \cdot e^{K_a (r_{lv} - r)} - \frac{K_b}{r^2} \quad (54), \qquad K = \frac{K_b \cdot K_a}{r_{lv}} \cdot e^{K_a (r_{lv} - r)} - \frac{K_b \cdot K_a}{r} \quad (55),$$

$$L = K_b \cdot K_a^2 \cdot e^{-K_a \cdot r} \cdot \int_{r_{lv}}^{r} \frac{e^{K_a \cdot r}}{r} dr \quad (56), \qquad M = V_{lv}^2 \cdot e^{K_a (r_{lv} - r)} \quad (57)$$

$$\text{and } K_a = -\frac{3}{4} \cdot \frac{C_d}{r_{part}} \frac{\rho_f}{\rho_p}, K_b = -\frac{a^2 \cdot \chi}{\mu_0} \tag{58}$$

on the other hand,

$$t = \int_r^{r_{lv}} \frac{dr}{V_r(r)} \tag{59}$$

z direction:

$$V_z(z) = \sqrt{K_1 + K_2 \cdot e^{K_3 (K_4 - z)}} \tag{60}$$

$$\text{where } K_1 = \frac{8}{3} \cdot g \cdot \frac{r_{part}}{C_d} \cdot \frac{\rho_p}{\rho_f}, K_2 = V_{z_{lv}}^2 - \frac{8}{3} \cdot g \cdot \frac{r_{part}}{C_d} \cdot \frac{\rho_p}{\rho_f}, K_3 = \frac{3}{4} \cdot \frac{C_d}{r_{part}} \cdot \frac{\rho_f}{\rho_p}, K_4 = z_{lv} \tag{61}$$

$$t = t_{lv} + N + O \tag{62}$$

$$\text{where } N = \frac{2 \cdot \sqrt{K_1 + K_2}}{K_3} - \frac{2 \cdot \sqrt{K_1 + K_2 \cdot e^{K_3 (K_4 - z)}}}{K_3} \quad (63), \quad \text{and}$$

$$O = \frac{2 \cdot \sqrt{K_1}}{K_3} \cdot Arctgh\left(\frac{\sqrt{K_1 + K_2 \cdot e^{K_3 (K_4 - z)}}}{\sqrt{K_1}} \right) - \frac{2 \cdot \sqrt{K_1}}{K_3} \cdot Arctgh\left(\frac{\sqrt{K_1 + K_2}}{\sqrt{K_1}} \right) \tag{64}$$

x direction (the direction tangential to the point where the particle departs - the direction of the $V_{\theta lv}$):

$$\frac{\partial^2 x}{\partial t^2} + \frac{q_1}{2} \cdot \left(\frac{\partial x}{\partial t^2} \right)^2 + f(t) \cdot (x - x_{lv}) = 0 \quad (65) \text{ with } f(t) = \frac{\frac{a^2 \cdot \chi}{\mu_0 \cdot r(t)^3} - \frac{3}{8} \cdot \frac{C_d}{r_{part}} \cdot \frac{\rho_f}{\rho_p} \cdot V_r(t)^2}{r(t)} \quad (66)$$

Equation (65) may be solved numerically.

Limiting conditions

We must first realise that the conditions for this section of the movement probably will be less limiting than the ones of the section a) and so the conditions of this section are always respected (theoretically) as long as the ones in section a) are.

- Derived from $\sqrt{k} \Rightarrow k \geq 0$ in r direction,

(SMC1A) $\begin{cases} J + K + L + M > 0 \text{ and knowing the other parameters } (\chi, a^2, \omega) \\ \text{we may implicitly obtain a condition for } r_{part}. \end{cases}$ (67)

OR

$$\text{(SMC1B)}\begin{cases} \dfrac{a^2\cdot\chi}{\mu_0} \text{ may have any value if } \dfrac{(J+K+L)}{-K_b}>0 & (68) \\ \dfrac{a^2\cdot\chi}{\mu_0}<\dfrac{-M}{\dfrac{(J+K+L)}{-K_b}} \text{ if } \dfrac{(J+K+L)}{-K_b}<0 & (69) \end{cases}$$

- Derived from $F_{Rr} > 0$ for the r direction,

$$\text{(SMC2A)}\begin{cases} P+\dfrac{K_a}{2}\cdot(J+K+L+M)>0 \text{ and knowing the other parameters } (\chi,\ a^2,\omega) \\ \text{we may implicitly obtain a condition for } r_{part}. \end{cases} \quad (70)$$

$$\text{where } P = -\frac{K_b}{r^3} \quad (71)$$

OR

$$\text{(SMC2B)}\begin{cases} \dfrac{a^2\cdot\chi}{\mu_0}>\dfrac{-\dfrac{K_a}{2}\cdot M}{-\dfrac{P}{K_b}-\dfrac{K_a}{2\cdot K_b}\cdot(J+K+L)} \quad \text{if} \quad -\dfrac{P}{K_b}-\dfrac{K_a}{2\cdot K_b}\cdot(J+K+L)>0 \end{cases} \quad (72)$$

- Derived from $\dfrac{F_{Rz}}{F_{Rr}} < tg\alpha$,

$$\text{(SMC3A)}\begin{cases} tg\alpha > \dfrac{g+\dfrac{K_a}{2}\cdot V_z^2}{-K_b+\dfrac{K_a}{2}\cdot V_r^2} \end{cases} \quad (73)$$

OR

$$\text{(SMC3B)}\begin{cases} (73) \text{ and knowing the other parameters } (\chi,\ a^2,\omega,\alpha) \\ \text{we may implicitly obtain a condition for } r_{part}. \end{cases} \quad (74)$$

OR

$$\text{(SMC3C)}\begin{cases} \dfrac{a^2\cdot\chi}{\mu_0}>\dfrac{\dfrac{\left(g+\dfrac{K_a}{2}\cdot V_{z_{iv}}^2\right)\cdot e^{-K_a\cdot(z_{iv}-z)}}{tg\alpha}-M}{-\dfrac{P}{K_b}-\dfrac{K_a}{2\cdot K_b}\cdot(J+K+L)} \quad \text{if} \quad \overbrace{-\dfrac{P}{K_b}-\dfrac{K_a}{2\cdot K_b}\cdot(J+K+L)}^{f(r_{part})}>0 \end{cases} \quad (75)$$

The limiting conditions presented above may present different limiting values for the same parameter, and when this happens we should choose the more limiting value as the correct one.

SIMULATION (10)

A complete simulation of the separation of a typical feed, will be presented at the conference. It is to be hoped that it will be compared with some results achieved in practice. Also the characteristics of the superconducting magnet generator will be presented.

OTHER CONFIGURATIONS (2)

Some other configurations are also being considered. As an example we refer to the configuration where the conical-trunk is fixed and the centrifugal effect is induced by a tangential feed that delivers the particles at the velocities needed (2). The mathematical model support of the theory has several differences relating to the one presented here.

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NOTATION

A - area resultant of the projection of the particle in the direction of movement
a - constant of magnetic field
B - magnetic field
Cd - drag coefficient
F_a - friction force
F_c - centrifugal force
F_d - drag force
F_{dr} - drag force on *r* axis
F_{dz} - drag force on *z* axis
F_{mag} - magnetic force
F_{Rr} - resultant force on *r* axis
F_{Rt} - resultant force on tangent direction
F_{Rz} - resultant force on *z* axis
g - Earth gravity constant
h_f - conical-trunk total height
h_i - observed initial height
I - current intensity flowing on the magnet
m - mass of the particle
N - reaction force made in the particle by the conical-trunk
P - weight force
r_a - conical-trunk's minimum radius
r_c - grid's maximum radius
r_f - conical-trunk's maximum radius
r_i - observed initial radius
r_{lv} - radius at which particle departs
r_{part} - particle's radius
r_R - "real" theoretical radius
T_{CG1} - theoretical coefficient global 1
T_{CG2} - theoretical coefficient global 2
t_{lv} - time gone until particle's departs from trunk
V_{olp} - particle's volume
V_p - particle's velocity
V_r - particle's velocity on *r* axis
V_{rlv} - particle's velocity on *r* axis at departure time
V_{rt} - particle's velocity on tangent direction
V_t - particle's velocity on tangent direction
V_z - particle's velocity on *z* axis
V_{zlv} - particle's velocity on *z* axe at departure time
V_θ - particle's velocity on θ axis
$V_{\theta lv}$ - particle's velocity on θ axis at departure time
z_{lv} - particle's height at departure
α - angle between the conical-trunk and the horizontal
χ - specific magnetic susceptibility
ΔS_{minor} - the minor of the spaces threaded on the tangential direction after one determined turn.
μ - friction coefficient
μ_0 - permeability of free space
ρ_f - specific weight of the fluid (medium)
ρ_p - specific weight of the particle
T - period of one turn
ω - angular velocity

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