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QUARK MODEL OF HADRONS

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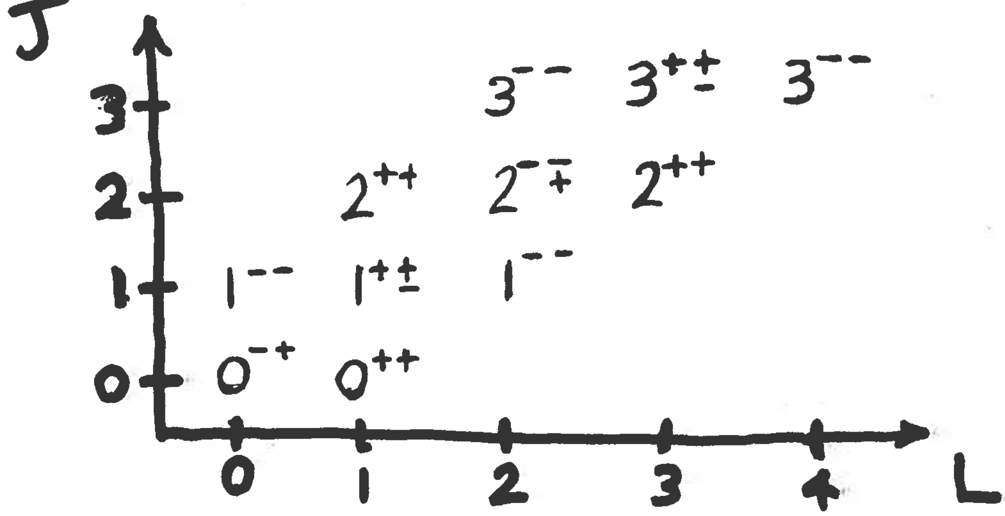
Talk given at the "Institut für Kernphysik, Technische Universität Wien", Vienna, July 1991

QUARK MODEL
(early 60's)

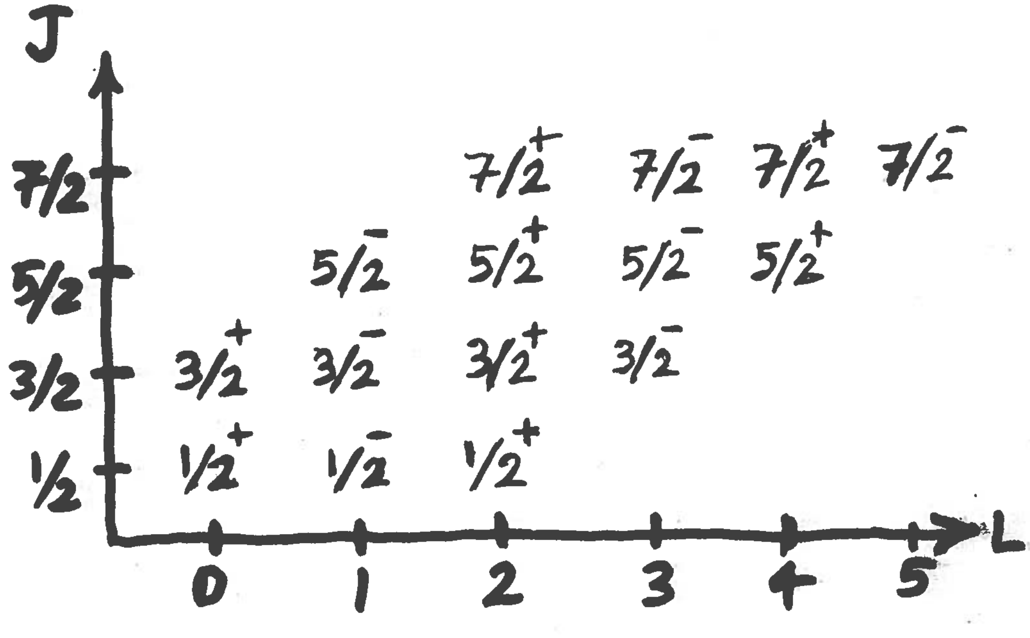
MESONS

$P(q\bar{q}, L, S) = (-1)^{L+1}$
$C(q\bar{q}) = (-1)^{L+S}$
$S = 0, 1$



BARYONS

$S(qqq) = 1/2, 3/2$
$P(qqq) = (-1)^L$



QUARK-MODEL ASSIGNMENTS FOR MESONS

$^{2S+1}L_J$	J^{PC}	$u\bar{d}, u\bar{u}, d\bar{d}$ $I=1$	$u\bar{u}, d\bar{d}, s\bar{s}$ $I=0$	$c\bar{c}$ $I=0$	$b\bar{b}$ $I=0$	$\bar{s}u, \bar{s}d$ $I=1/2$	$c\bar{u}, c\bar{d}$ $I=1/2$	$c\bar{s}$ $I=0$	$\bar{b}u, \bar{b}d$ $I=1/2$
$^1S_0$	0^{-+}	π	η, η'	η_c		K	D	D_s	B
$^3S_1$	1^{--}	ρ	ϕ, ω	J/ψ	Υ	$K^*(892)$	$D^*(2010)$		
$^1P_1$	1^{+-}	$b_1(1235)$	$h_1(1170)$			K_{1B}	$D_1(2420)$	$D_{s1}(2536)$	
$^3P_0$	0^{++}	$a_0(980)$	$f_0(975), f_0(1400)$	$\chi_{c0}(1P)$	$\chi_{b0}(1P)$	$K_0^*(1430)$			
$^3P_1$	1^{++}	$a_1(1260)$	$f_1(1285), f_1(1420)$	$\chi_{c1}(1P)$	$\chi_{b1}(1P)$	K_{1A}			
$^3P_2$	2^{++}	$a_2(1320)$	$f_2'(1525), f_2(1270)$	$\chi_{c2}(1P)$	$\chi_{b2}(1P)$	$K_2^*(1430)$	$D_2^*(2460)$		
$^1D_2$	2^{-+}	$\pi_2(1670)$							
$^3D_1$	1^{--}			$\psi(3770)$					
$^3D_2$	2^{--}					$K_2(1770)$			
$^3D_3$	3^{--}	$\rho_3(1690)$	$\omega_3(1670)$			$K_3^*(1780)$			

From: "Review of Particle Properties", 1990

QUARK-MODEL ASSIGNMENTS FOR BARYONS

J^P	$(D.L^P_N)$	S	Octet members				Singlets
$1/2^+$	(56.0^+_0)	1/2	$N(939)$	$\Lambda(1116)$	$\Sigma(1193)$	$\Xi(1318)$	
$1/2^+$	(56.0^+_2)	1/2	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$\Xi(?)$	
$1/2^-$	(70.1^-_1)	1/2	$N(1535)$	$\Lambda(1670)$	$\Sigma(1620)$	$\Xi(?)$	$\Lambda(1405)$
$3/2^-$	(70.1^-_1)	1/2	$N(1520)$	$\Lambda(1690)$	$\Sigma(1670)$	$\Xi(1820)$	$\Lambda(1520)$
$1/2^-$	(70.1^-_1)	3/2	$N(1650)$	$\Lambda(1800)$	$\Sigma(1750)$	$\Xi(?)$	
$3/2^-$	(70.1^-_1)	3/2	$N(1700)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^-$	(70.1^-_1)	3/2	$N(1675)$	$\Lambda(1830)$	$\Sigma(1775)$	$\Xi(?)$	
$1/2^+$	(70.0^+_2)	1/2	$N(1710)$	$\Lambda(1810)$	$\Sigma(1880)$	$\Xi(?)$	$\Lambda(?)$
$3/2^+$	(56.2^+_2)	1/2	$N(1720)$	$\Lambda(1890)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^+$	(56.2^+_2)	1/2	$N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$	$\Xi(2030)$	
$7/2^-$	(70.3^-_3)	1/2	$N(2190)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	$\Lambda(2100)$
$9/2^-$	(70.3^-_3)	3/2	$N(2250)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$9/2^+$	(56.4^+_4)	1/2	$N(2220)$	$\Lambda(2350)$	$\Sigma(?)$	$\Xi(?)$	

J^P	$(D.L^P_N)$	S	Decuplet members			
$3/2^+$	(56.0^+_0)	3/2	$\Delta(1232)$	$\Sigma(1385)$	$\Xi(1530)$	$\Omega(1672)$
$1/2^-$	(70.1^-_1)	1/2	$\Delta(1620)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$
$3/2^-$	(70.1^-_1)	1/2	$\Delta(1700)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$
$5/2^+$	(56.2^+_2)	3/2	$\Delta(1905)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$
$7/2^+$	(56.2^+_2)	3/2	$\Delta(1950)$	$\Sigma(2030)$	$\Xi(?)$	$\Omega(?)$
$11/2^+$	(56.4^+_4)	3/2	$\Delta(2420)$	$\Sigma(?)$	$\Xi(?)$	$\Omega(?)$

SU(6) flavor-spin basis

From: "Review of Particle Properties", 1990

RELATIVISTIC, CHIRAL QUARK MODEL

(1984, Le Yaouanc, Oliver, Pène & Raynal)

$$H = \int d^3x \Big[\psi^\dagger(x)(m\beta - i\bar{\alpha}\cdot\bar{\nabla})\psi(x) + \frac{1}{2}\int d^3y \, V(\bar{x}-\bar{y})\, \psi^\dagger(x)\frac{\lambda^a}{2}\psi(x)\, \psi^\dagger(y)\frac{\lambda^a}{2}\psi(y) \Big]$$

- Effective, instantaneous q–$q(\bar{q})$ interaction
- **In the limit $m \to 0$, H is chiral invariant.**
- This type of interaction leads to color confinement.

The Interaction Picture

$$H = H_0 + H_I$$

$$\begin{cases} i\dfrac{\partial \psi(x)}{\partial t} = [\psi(x), H_0] \\ i\dfrac{\partial |\Omega\rangle}{\partial t} = H_I(t)|\Omega\rangle \end{cases}$$

The choice of H_0, H_I is arbitrary. However, different schemes may give results that may be inconvenient.

Example: $H_0 = H_{\text{Dirac (free)}} \Rightarrow$ the vacuum does not have minimum E!

Once H_0 is chosen, the field operator can be derived.

$$\psi(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \left[u_s(\bar{p}) \underset{\substack{\uparrow \\ \text{quark} \\ \text{annihilation}}}{b_s(\bar{p})} + v_s(\bar{p}) \underset{\substack{\uparrow \\ \text{antiquark} \\ \text{creation}}}{d_s^{\dagger}(-\bar{p})} \right] e^{-ip\cdot x}$$

u_s, v_s are the solution to certain equat. determined by the choice of H_0.

Let us try to find H_0 that leads to a vacuum with a minimum energy: Rewrite H as

$$H = E_{vac} + H_0 + H_I$$

$$H_0 = \int d^3p : \psi^{\dagger}(\bar{p}) \left[A(p)\beta + B(p)\bar{\alpha}\cdot\hat{p} \right] \psi(\bar{p}):$$

$$H_I = \frac{1}{2}\int d^3p\, d^3k\, d^3q\, V(\bar{q}) : \psi^{\dagger}(\bar{p}+\bar{q}) \frac{\lambda^a}{2} \psi(\bar{p})\, \psi^{\dagger}(\bar{k}-\bar{q}) \frac{\lambda^2}{2} \psi(\bar{k}):$$

Which leads to the equations:

$$\begin{cases} \left[A(p)\beta + B(p)\,\hat{p}\cdot\bar{\alpha} \right] u_s(\bar{p}) = E(p)\, u(p) \\ \left[A(p)\beta + B(p)\,\hat{p}\cdot\bar{\alpha} \right] v_s(\bar{p}) = -E(p)\, v(p) \end{cases}$$

$$E^2 \equiv A^2 + B^2$$

The solution to these equations is of the form:

$$A(p) = m + \frac{2}{3}\int d^3k \; V(\bar{p}-\bar{k}) \sin \varphi(p)$$

$$B(p) = p + \frac{2}{3}\int d^3k \; V(\bar{p}-\bar{k})(\hat{k}\cdot\hat{p}) \cos \varphi(p)$$

(u_s, v_s certain functions of $\varphi(p)$)

The function $\varphi(p)$ is up to now arbitrary. **It governs the dynamics of the system and we call it CHIRAL ANGLE.**

VACUUM CONDENSATE

Minimization of $E_{vac.}$ gives an equation that defines the chiral angle

$$\boxed{A(p)\cos\varphi(p) - B(p)\sin\varphi(p) = 0}$$

MASS-GAP Equation

In terms of "free-Dirac quarks" the new vacuum is a condensate of scalar $q\bar{q}$ pairs.

$\Downarrow$

Spontaneous breaking of the chiral symmetry.

HARMONIC OSCILLATOR CONFINEMENT

$$V(x) = -K_0^3 x^2 \qquad (K_0 > 0)$$

$$V(\bar{q}) = K_0^3 \nabla^2 \delta(\bar{q})$$

- The mass-gap equation becomes a diff. equation that can be easily solved in terms of m and K_0

- For each flavor there is one chiral angle and one mass-gap equation.

- If $m \to \infty$, $\varphi(p) \to \pi/2$

  If $m = 0$, $K_0 = 0$ (chiral symmetric vacuum)
  $\varphi(p) \to 0$

THE SALPETER EQUATION

$$[M - E(k) - \bar{E}(k)]\,\phi^{L}_{s_1 s_2}(\vec{k}) = -\frac{4}{3}\int d^3p\, V(\vec{k}-\vec{p}) \times$$

$$\times\Big[u^{+}_{s_1}(\vec{k})\,u_{s_3}(\vec{p})\,\bar{v}^{+}_{s_4}(\vec{p})\,\bar{v}_{s_2}(\vec{k})\,\phi^{L}_{s_3 s_4}(\vec{p})$$

$$+\,u^{+}_{s_1}(\vec{k})\,\bar{v}_{s_4}(\vec{p})\,u^{+}_{s_3}(\vec{p})\,\bar{v}_{s_2}(\vec{k})\,\phi^{S}_{s_3 s_4}(\vec{p})\Big]$$

For the ρ,

$$\left[\left(\frac{d^2}{dk^2} - 2E - \frac{8\,g^2(k)}{3k^2}\right)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + M\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} - \frac{\varphi'^2(k)}{6}\begin{pmatrix}3 & 1\\ 1 & 3\end{pmatrix} + \frac{\cos^2\varphi(k)}{3k^2}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\right]\begin{pmatrix}\nu_L\\ \nu_S\end{pmatrix} = 0\,.$$

For the π,

$$\left[\left(\frac{d^2}{dk^2} - 2E\right)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + M\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} - \left(\frac{\varphi'^2(k)}{2} + \frac{\cos^2\varphi(k)}{k^2}\right)\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\right]\begin{pmatrix}\nu_L\\ \nu_S\end{pmatrix} = 0$$

The transition potential between Large and small channels is

$$\frac{\varphi'^2(k)}{2} + \frac{\cos^2\varphi(k)}{k^2} \qquad \text{(for the } \pi)$$

and 1/3 of that for the ρ.

In the chiral limit, $m \to 0$,

$$\nu_L = \nu_S \qquad \text{(for the } \pi)$$

and $\boxed{M_\pi = 0}$

BOUND STATES

$$H|\phi\rangle_L = M|\phi\rangle_L$$

Mesons:

$$|\phi_m\rangle = \int d^3p_1\, d^3p_2\; \delta(\bar{p}_1+\bar{p}_2)\, \phi(\bar{p}_1,\bar{p}_2)\, \chi(1,2)\, b_c^\dagger(\bar{p}_1)\, d_c^\dagger(\bar{p}_2)\, |0\rangle$$

$$\left[-\frac{d^2}{dk^2} + E(k) + \bar{E}(k) + \frac{L(L+1)}{k^2} + C.T. + S\text{-}S + S\text{-}L + T\right]\phi(k) = M\phi(k)$$

Baryons:

$$|\phi_B\rangle = \int d^9p\; \delta(\bar{p}_1+\bar{p}_2+\bar{p}_3)\, \phi(\bar{p}_1,\bar{p}_2,\bar{p}_3)\, \chi(1,2,3)\, \epsilon_{c_1c_2c_3}\, b_{c_1}^\dagger(\bar{p}_1)\, b_{c_2}^\dagger(\bar{p}_2)\, b_{c_3}^\dagger(\; |0\rangle$$

Force	Mesons	Baryons
Spin-spin	$-\dfrac{4}{3k^2} g^2(k)\, \bar{g}^2(k)\, \mathbf{S}_1 \cdot \mathbf{S}_2$	$\dfrac{\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2}{3p_1p_2} g^2(p_1)\, g^2(p_2)\, \mathbf{S}_1 \cdot \mathbf{S}_2$
Spin-orbit	$\dfrac{2}{k^2}\left[g^2(k)\, \mathbf{S}_1 + \bar{g}^2(k)\, \mathbf{S}_2\right] \cdot \mathbf{L}$	$-\dfrac{6g^2(p_1)}{p_1^2} \mathbf{S}_1 \cdot \mathbf{L}_1 + \dfrac{3g^2(p_1)}{p_1^2} (\mathbf{S}_1 + \mathbf{S}_2) \cdot (i\nabla_{p_2} \times \mathbf{p}_1)$
Tensor	$\dfrac{2}{k^2} g^2(k)\, \bar{g}^2(k)$ $[(\hat{\mathbf{k}} \cdot \mathbf{S}_1)(\hat{\mathbf{k}} \cdot \mathbf{S}_2) - \frac{1}{3}\, \mathbf{S}_1 \cdot \mathbf{S}_2]$	$\dfrac{3\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2}{p_1p_2} g^2(p_1)\, g^2(p_2) [(\hat{\mathbf{p}}_1 \cdot \mathbf{S}_1)(\hat{\mathbf{p}}_2 \cdot \mathbf{S}_2)$ $-\frac{1}{3}(\hat{\mathbf{p}}_1 \cdot \hat{\mathbf{p}}_2)(\mathbf{S}_1 \cdot \mathbf{S}_2)]$

$$g^2(k) \equiv 1 - \sin\varphi(k)$$

CHARM-SECTOR MESON SPECTRUM

Meson	J^{PC}	${}^{S}L_J$	Theory (MeV)	Experiment (MeV)
η_c	0^{-+}	${}^1S_0$	3096	2979
J/ψ	1^{--}	${}^3S_1 + {}^3D_1$ [a]	3097	3097
χ_{c_0}	0^{++}	${}^3P_0$	3332	3415
χ_{c_1}	1^{++}	${}^3P_1$	3343	3511
χ_{c_2}	2^{++}	${}^3P_2 + {}^3F_2$ [a]	3365	3556
ψ'	1^{--}	${}^3S_1 + {}^3D_1$ [a]	3579	3686
ψ''	1^{--}	${}^3S_1 + {}^3D_1$ [a]	3611	3770
ψ'''	1^{--}	${}^3S_1 + {}^3D_1$ [a]	4155	4040
ψ''''	1^{--}	${}^3S_1 + {}^3D_1$ [a]	4209	4159
ψ'''''	1^{--}	${}^3S_1 + {}^3D_1$ [a]	4935	4915
D	0^-	${}^1S_0$	1998	1869
D_0^*	0^+	${}^3P_0$	2216	—
D^*	1^-	${}^3S_1$	2005	2007
D_1	1^+	${}^3P_1 + {}^1P_1$ [b]	2271	—
D_1	1^+	${}^3P_1 + {}^1P_1$ [b]	2499	2424
D_2^*	2^+	${}^3P_2 + {}^3F_2$ [a]	2552	2459

$\left(\frac{4}{3}\right)^{1/3} K_0 = 290$ MeV

$m_c = 1362$ MeV

$m_u = m_d = 0$

MASSES

	Theory (MeV)	Experiment (MeV)
N	1378	940
Δ	1612	1232
ρ	742	770
δ	778	980
A_1	1183	1270
B	1348	1235

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COLOR CONFINEMENT

Consider,

$$V(x) = -K_0^3 x^2 + U$$

$$V(q) = K_0^3 \nabla^2 \delta(q) + U\delta(q)$$

As $U \to \infty$, (with $K_0, U > 0$ and $\lim_{x \to \pm\infty} V(x) = 0$)

① The kinetic energy of a quark (antiquark) becomes infinite. $\frac{2}{3}U$

② The mass gap equation is independent of U.

③ Bound state equations for hadrons are also independent of the potential shift. The contributions of U from kinetic and potential energies cancel. $\frac{4}{3}U$

④ For any system of N quarks (antiquarks), the kinetic energy, arising from U is

$$E_{kin.} = \frac{2}{3} N U$$

while for the interaction energy.

$$\sum_{i<j} \frac{\lambda_i \cdot \lambda_j}{4} U = \left[\frac{1}{8}\Lambda\cdot\Lambda - \frac{2}{3}N\right] U$$

$$E = \frac{1}{8}\Lambda\cdot\Lambda\, U \quad (\Lambda \text{ is the total color})$$

$E \to \infty$ for any non color singlet.
and color non-singlets are not allowed

CONCLUSIONS

- Confinement and dynamical breaking of chiral symmetry are both related and can be understood in terms of an effective q-q ($\bar{q}$) interaction.

- The spin-spin, spin-orbit and tensor interactions in Hadrons are all functions of the chiral angle $\varphi(p)$ which measures the amount of vacuum condensation.

- Masses of Pseudo-Goldstone mesons, heavier mesons and baryons (N, Δ) can be explained with an effective quark theory.

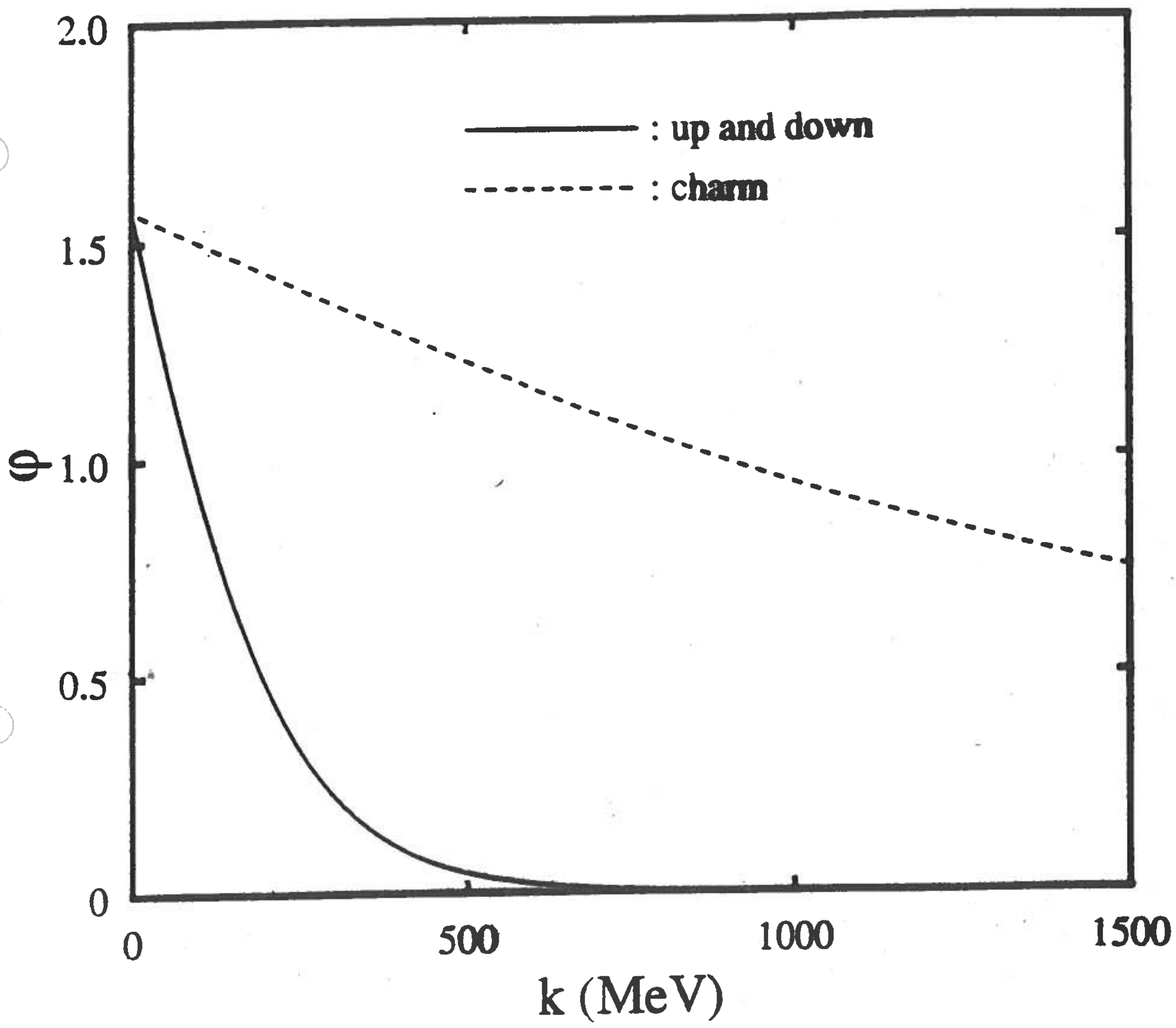
PROBLEMS.

- A more realistic confining potential such as a linear potential should be used.

- The masses of N-Δ should decrease 300-400 MeV as the result of coupling to mesons.

THE CHIRAL ANGLE $\varphi(k)$

(Harmonic oscillator potential)

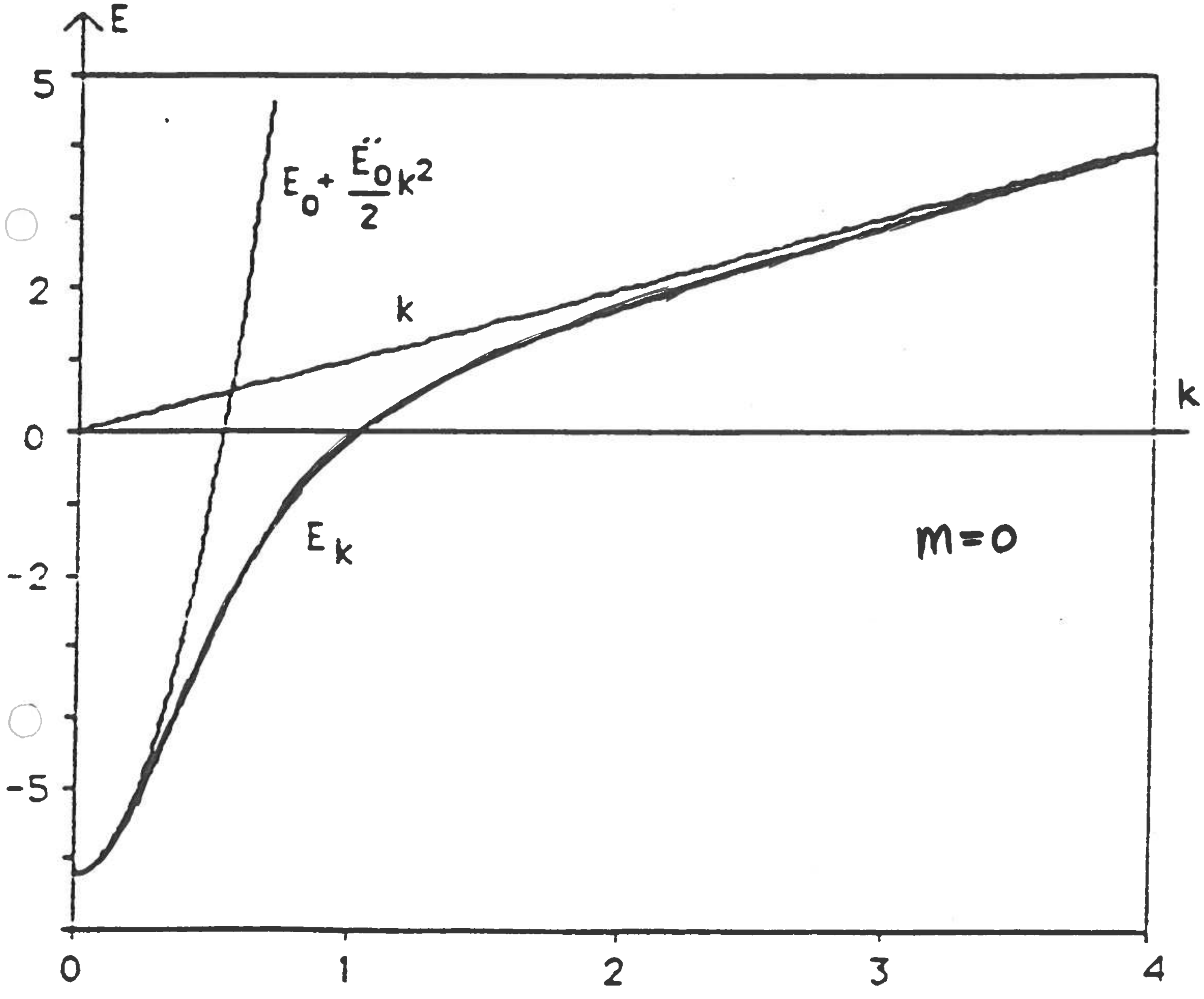


$\left(\frac{4}{3}\right)^{1/3} K_0 = 290$ MeV

$m_c = 1362$ MeV

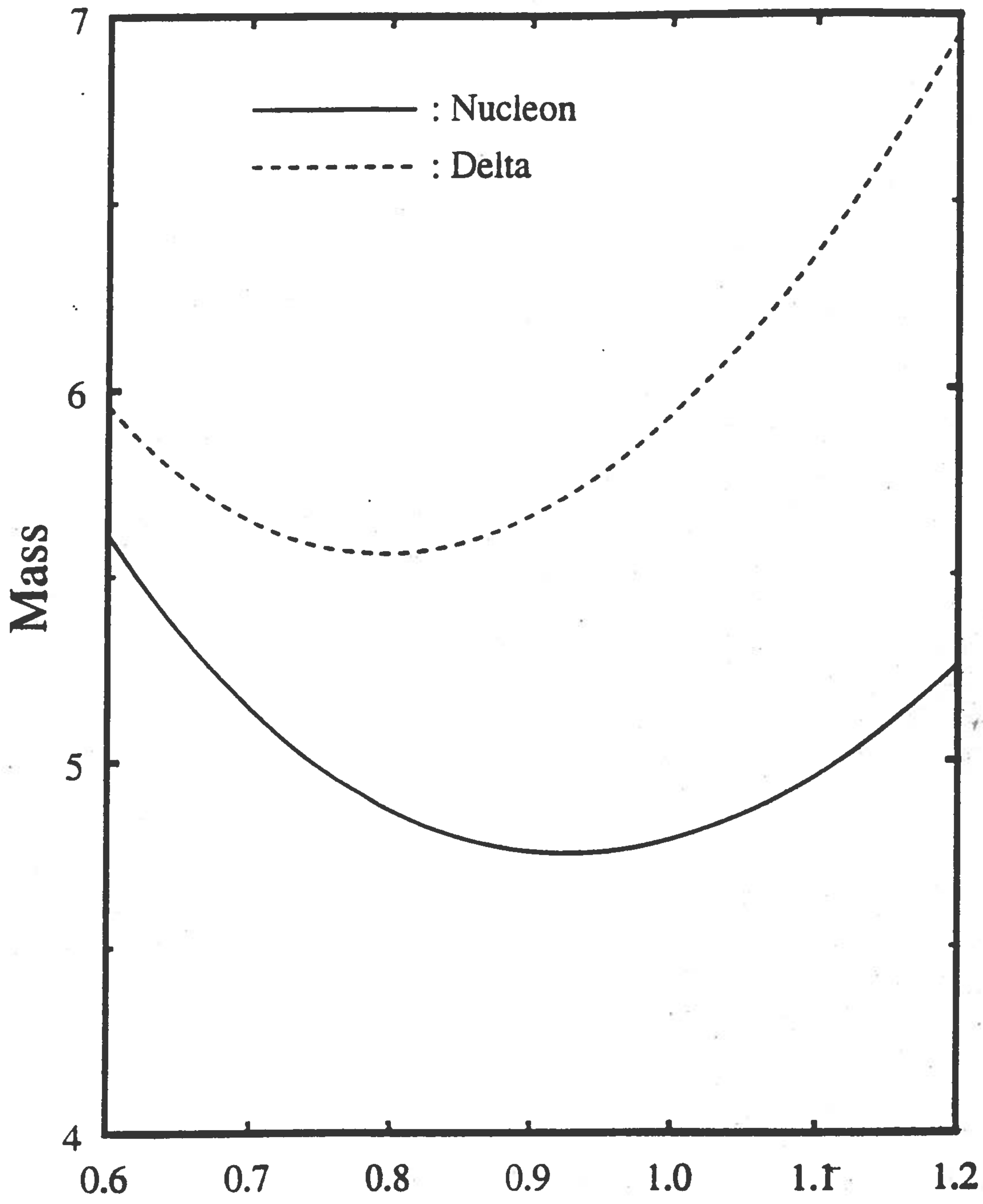
$m_u = m_d = 0$

QUARK ENERGY E(k)
(Harmonic oscillator potential)



Bicudo & Ribeiro, Phys. Rev. D42, 1611 (1990)

MASSES OF THE NUCLEON AND THE Δ

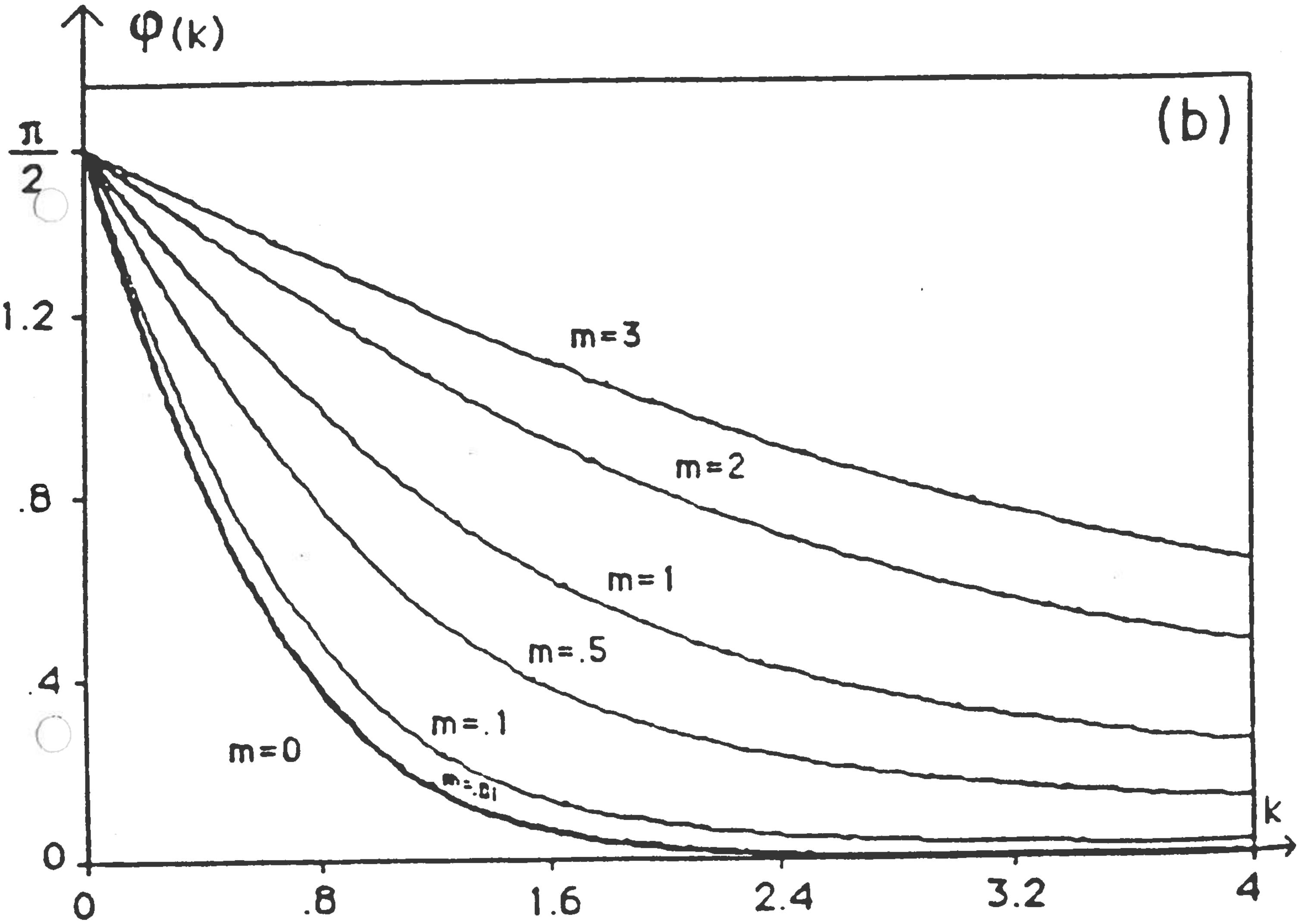


(variational parameter)

M, α in units of $\left(\frac{4}{3}\right)^{1/3} K_0$

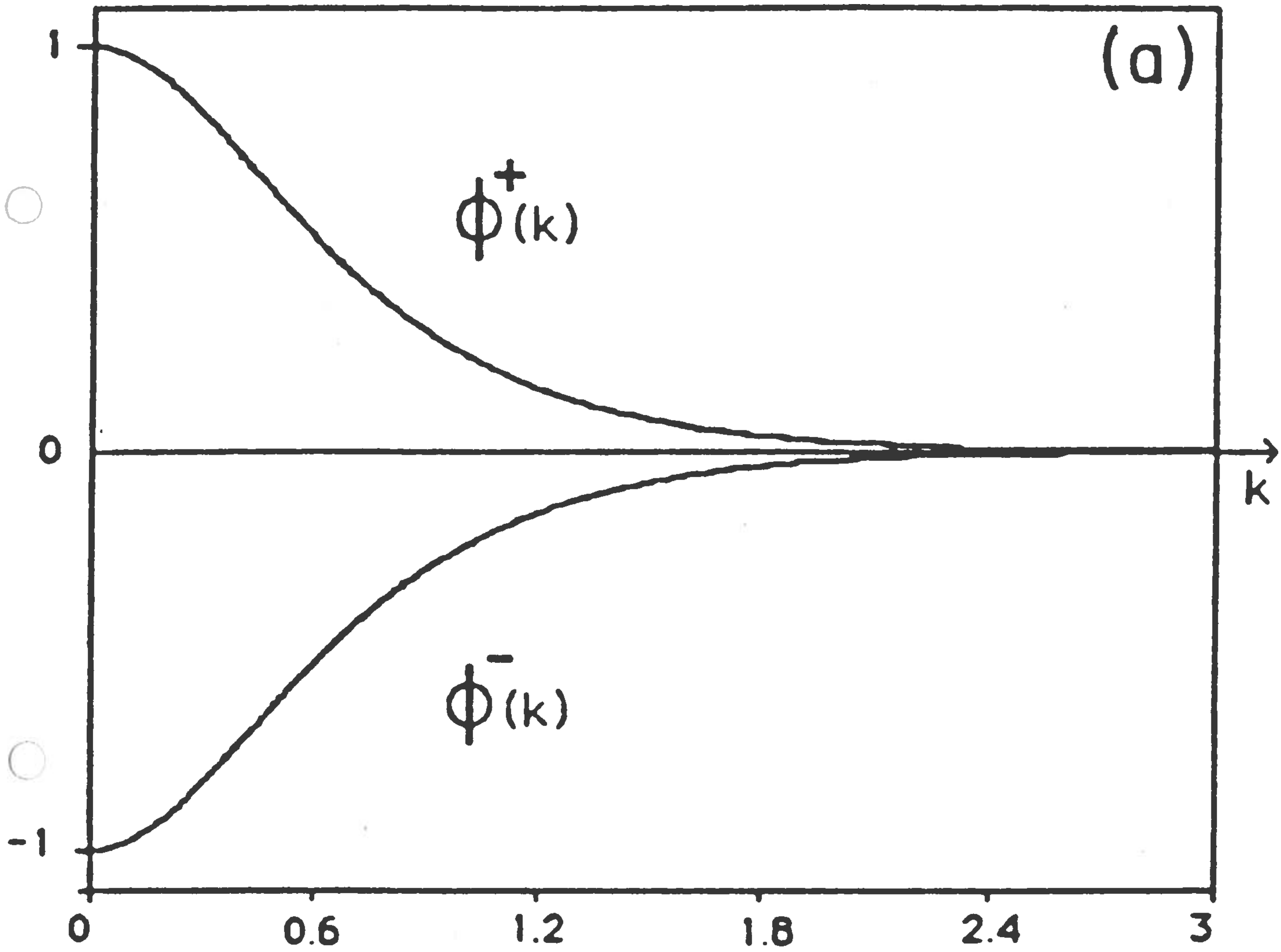
SOLUTION TO THE MASS-GAP EQUATION

(Harmonic oscillator potential)



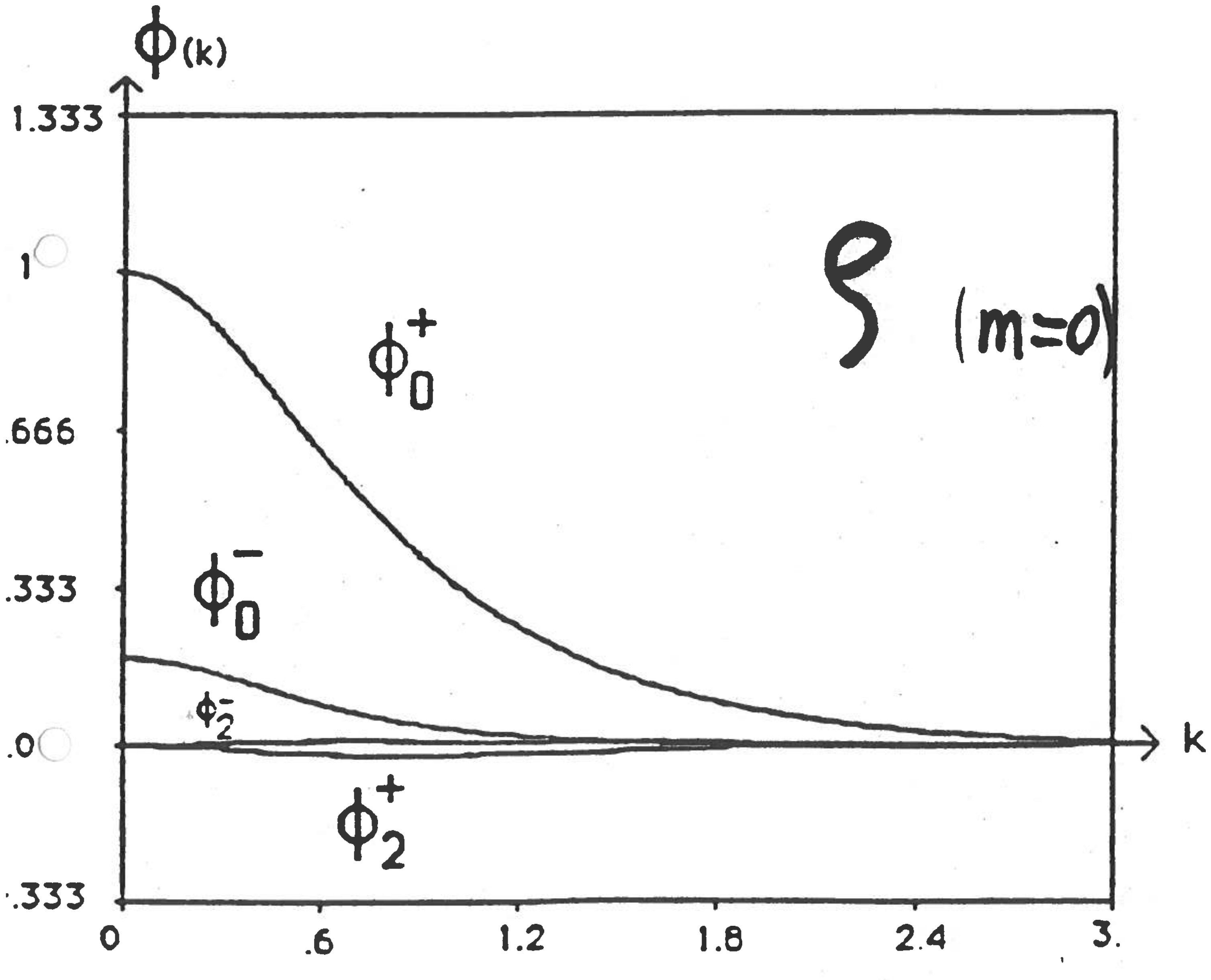
Bicudo & Ribeiro, Phys. Rev. D**42**, 1611 (1990)

PION'S WAVE FUNCTION
(Ground state)



Bicudo & Ribeiro, Phys. Rev. D42, 1625 (1990)

WAVE FUNCTION OF THE ρ



Bicudo & Ribeiro, Phys. Rev. D42, 1625 (1990)